FLORENTIN SMARANDACHE

SUBSET SEMIRINGS

Subset Semirings

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PREFACE

In this book authors study the new notion of the algebraic structure of the subset semirings using the subsets of rings or semirings. This study is innovative and interesting for the authors feel giving algebraic structure to collection of sets is not a new study, for when set theory was introduced such study was in vogue. But a systematic development of constructing algebraic structures using subsets of a set is absent, except for the set topology and in the construction of Boolean algebras.

The authors have explored the study of constructing subset algebraic structures like semigroups, groupoids, semirings, non commutative topological spaces, non associative topological spaces, semivector spaces and semilinear algebras.

We have constructed semirings using rings of both finite and infinite order. Thus using finite rings we are in a position to construct infinite number of finite semirings both commutative as well as non commutative.

It is important to keep on record we have mainly distributive lattices of finite order which contribute for finite semirings. However this new algebraic structure helps to give several finite semirings. This is the advantage of using this new algebraic structure.

We call those subset semirings constructed using rings as subset semirings of type I and when we use semirings in the place of rings we call those subset semirings as subset semirings of type II.

Several interesting properties about substructures and special elements are studied and discussed in this book. We have subset zero divisors, subset idempotents and subset nilpotents.

We further state these structures find their applications in those places where semirings and lattices find their applications.

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Chapter One

INTRODUCTION

In this chapter we introduce only the basic concepts used in this book. In most cases we only give references of them.

We use only distributive lattices for finite semirings. For more about distributive lattices please refer [1].

The notion of semirings and Smarandache semirings can be had from [14]. We use the concept of subset semiring. These concepts are very recently introduced in [25-6].

Study of subsets of any algebraic structure and inducing the same operation on these subsets can maximum give a semiring in case of algebraic structures with two binary operations and semigroup in case of algebraic structures with one binary operation. We take in this book, only algebraic structures with operations which are associative.

Here we study only subset semirings, which are the subsets either from a ring or a semiring. Define these subset semirings as type I (when a ring is used) and subset semirings as type II when a semiring is used. We give many examples to describe various properties enjoyed by these new algebraic structures.

Further we also introduce the notion of subset zero divisors, subset units, subset idempotents and subset nilpotents.

We also describe the Smarandache analogue. In this book we study the substructure of the subset semirings and the Smarandache analogue.

Finally we study the subset set semiring ideals of these subset semirings. On every subset semiring we can define four topologies on S, T_o, T_s, $T_\cup$ and $T_\cap$. The cardinality of T_s and $T_\cup$ are the same as that of S and that of T_o and $T_\cap$ have one element more in S.

That is $o(T_s) = o(T_\cup) = o(S)$ and $o(T_\cap) = o(T_o) = o(S) + 1$.

We also see in case S is finite the notion of tree can be defined for T_s, T_o, $T_\cup$ and $T_\cap$. These trees can find applications in computer science.

We introduce the notion of special set ideal semiring topological subset semiring spaces over a subsemiring P_1; we have $T_o^{P_1}$, $T_\cup^{P_1}$, $T_\cap^{P_1}$ and $T_S^{P_1}$ to be the four new types of topological spaces.

We see using W_1 = {Collection of all subsets from the subsemiring P_1} $\subseteq$ S, the subset subsemiring of S. $T_o^{W_1}$, $T_\cup^{W_1}$, $T_\cap^{W_1}$ and $T_S^{W_1}$ are the special strong new set semiring ideal topological subset semiring spaces defined over the subset subsemiring W_1 of S.

In case S is finite we have with these four types of trees associated with the topological spaces of finite order. Interested reader can find applications of these trees.

Chapter Two

SUBSET SEMIRINGS OF TYPE I

In this chapter we for the first time introduce the notion of subset semirings of type I using rings. This study is both innovative and interesting. We see we cannot get using subsets of a ring, a subset ring, the maximum structure we can get is only a subset semiring which we call as type I subset semiring. We study subset zero divisors, subset idempotents, subset ideals and subset subsemirings of these subset semirings.

We recall the definition of this concept in the following:

DEFINITION 2.1: *Let R be a ring.*
S = {Collection of all subsets of a ring}. S under the operations of R is a semiring known as the subset semiring of type I of the ring R.

We will give a few examples.

Example 2.1: Let S = {Collection of all subsets of the ring Z_4} be the subset semiring of the ring Z_4. The operations on S are performed as follows:

For A = {2, 3, 0} and B = {1, 0, 2} in S.

We see

$$A + B = \{2, 3, 0\} + \{1, 0, 2\}$$

$$= \{2 + 1, 2 + 0, 2 + 2, 3 + 1, 3 + 0, 3 + 2, 0 + 1, 0, 0 + 2\}$$

$$= \{3, 2, 0, 1\} \in S.$$

$$A \times B = \{2, 3, 0\} \times \{1, 0, 2\}$$

$$= \{2 \times 1, 2 \times 0, 2 \times 2, 3 \times 0, 3 \times 1, 3 \times 2, 0 \times 1, 0 \times 0, 0 \times 2\}$$

$$= \{2, 0, 3\} \in S.$$

Thus (S, +, ×) is only subset semiring and not a ring for every A we do not have –A such that A + (–A) = {0}.

For take A = {0, 2, 1} and –A = {0, 2, 3} ∈ S;
but

$$A + (–A) = \{0, 2, 1\} + \{0, 2, 3\}$$

$$= \{0 + 0, 2 + 0, 1 + 0, 0 + 2, 2 + 2, 1 + 2, 0 + 3, 2 + 3, 1 + 3\}$$

$$= \{0, 2, 1, 3\} \neq \{0\}.$$

Hence S can only be a subset semiring. By this method we get infinite collection of finite semirings.

Clearly S is not a subst semifield for we have A ∈ S with A + A = {0}.

For take A = {2} $\in$ S we see

$$\begin{aligned} A + A &= \{2\} + \{2\} \\ &= \{2 + 2\} = \{0\} \in S. \end{aligned}$$

$$A \times A = \{2\} \times \{2\} = \{2 \times 2\} = \{0\} \in S.$$

Thus S has zero divisors.

Hence this S cannot be a subset strict semiring, however S is a commutative subset semiring.

Example 2.2: Let
S = {Collection of all subsets from the field Z_7} be the subset semiring of Z_7. S has no subset zero divisors. S has subset units but S is only a subset semiring and not a subset semifield.

For take A = {6} $\in$ S.

A × A = {6} × {6} = {1} is a subset unit of S.

We see for A, B $\in S \setminus \{0\}$; $A \times B \neq \{0\}$.

However for every A $\in$ S we do not have a B such that A + B = {0} so S is not a subset ring only a subset semiring.

Let A = {3, 4, 0, 2} and B = {4, 3, 5} $\in$ S

$$\begin{aligned} A + B &= \{3, 4, 0, 2\} + \{4, 3, 5\} \\ &= \{3 + 4, 3 + 3, 3 + 5, 4 + 4, 4 + 3, 4 + 5, 0 + 4, \\ &\quad 0 + 3, 0 + 5, 2 + 4, 2 + 3, 2 + 5\} \\ &= \{0, 6, 1, 2, 4, 3, 5\} \in S. \end{aligned}$$

Now

$$A \times B = \{3, 4, 0, 2\} \times \{4, 3, 5\}$$

$$= \{3 \times 4, 3 \times 3, 3 \times 5, 4 \times 4, 4 \times 3, 4 \times 5, 0 \times 4, 0 \times 3, 0 \times 5, 2 \times 4, 2 \times 3, 2 \times 5\}$$

$$= \{5, 2, 1, 6, 0, 3\} \in S.$$

$$A \times A = \{3, 4, 0, 2\} \times \{3, 4, 0, 2\}$$

$$= \{3 \times 3, 4 \times 3, 0 \times 3, 2 \times 3, 3 \times 4, 4 \times 4, 0 \times 4, 2 \times 4, 3 \times 0, 4 \times 0, 0 \times 0, 2 \times 0, 3 \times 2, 4 \times 2, 0 \times 2, 2 \times 2\}$$

$$= \{2, 5, 0, 6, 4, 1\} \in S.$$

This is the way operations are performed.
Take $\{5\} = A \in S$.

We see $A \times A = \{5\} \times \{5\} = \{4\}$.
$A \times A \times A = \{5\} \times \{4\} = \{6\}$.
$A \times A \times A \times A = \{6\} \times \{5\} = \{2\}$.
$A \times A \times A \times A \times A = \{2\} \times \{5\} = \{3\}$ and
$A \times A \times A \times A \times A \times A = \{3\} \times \{5\} = \{1\} \in S$.

Thus $A^6 = \{1\}$ is a element of order 6.

Example 2.3: Let S = {Collection of all subsets from the neutrosophic ring $R = \langle Z_{12} \cup I\rangle$} be the subset semiring. S is a subset neutrosophic semiring of type I.

Example 2.4: Let S = {Collection of all subsets from the complex modulo integer ring $R = C(Z_{10})$} be the subset semiring.

S is a subset complex modulo integer semiring of the ring R of type I.

Example 2.5: Let S = {Collection of all subsets from the finite complex number neutrosophic ring $R = C(\langle Z_{15} \cup I\rangle)$} be the subset semiring of the ring R. S is a finite complex number neutrosophic subset semiring.

All the five examples are examples of finite subset semirings of type I.

Now we give examples of subset semirings of infinite order.

Example 2.6: Let
S = {Collection of all subsets from the ring R = Z} be the subset semiring. S is an infinite subset semiring which has no subset zero divisors or subset units.

Let A = {3, 5, 8, –5, 1} and B = {8, –1, 9, –10} $\in$ S.

A + B = {3, 5, 8, –5, 1} + {8, –1, 9, –10}

= {3 + 8, 5 + 8, 8 + 8, –5 + 8, 1 + 8, 3 – 1, 5 – 1,
8 – 1, –5 – 1, 1 – 1, 3 + 9, 5 + 9, 8 + 9, –5 + 9,
1 + 9, 3 – 10, 5 – 10, 8 – 10, –5 – 10, 1 – 10}

= {11, 13, 16, 3, 9, 2, 4, 7, –6, 0, 12, 14, 17, 10, –7,
–5, –2, –15, –9} $\in$ S.

We see if A = {5} and B = {–5} in S then A + B = {0}.

However for every A $\in$ S we do not have a B (= –A) such that A + B = {0}.

We see A × B = {3, 5, 8, –5, 1} × {8, –1, 9, –10}

= {3 × 8, 3 ×–1, 3 ×9, 3 × –10, 5 × 8, 5 ×–1,
5 × 9, 5 × –10, 8 × 8, 8 × –1, 8 × 9, 8 × –10,
–5 × 8, –5 × –1, –5 × 9, –5 × –10, 1 × 8,
1 × –1, 1 × 9, 1 × –10}

= {24, –3, 27, –30, 40, –5, 45, –1, –50, 64, –8, 72,
–80, –40, 5, –45, 50, 8, 9, –10} $\in$ S.

We see A × B = {0} is not possible for A, B ∈ S \ {0}.

Example 2.7: Let
S = {Collection of all subsets from the ring ⟨Q ∪ I⟩} be the subset semiring of infinite order of the ring ⟨Q ∪ I ⟩. This subset semiring has infinite number of subset zero divisors.

For take A = {5 – 5I} and B = {3I} ∈ S.

We see A × B = {5 –5I} × {3I}
= {5–5I × 3I}
= {15I – 15I}
= {0}, is a subset zero divisor of S.

Clearly S has infinite number of subset zero divisors given by A = {3 – 3I, 8 – 8I, 9I – 9} and B = {2I} ∈ S.

We see

A × B = {3 – 3I, 8 – 8I, 9I – 9} × {2I}
= {3 – 3I × 2I, 8 – 8I × 2I, 9I – 9× 2I }
= {6I – 6I , 16I – 16I, 18I – 18I}
= {0}.

Take A = {5 – 5I, 7 – 7I, 18 – 18I} and B = {2I, I} ∈ S.

We see

A × B = {5 – 5I, 7 – 7I, 18 – 18I} × {2I, I}
= {5 – 5I × 2I, 7 – 7I × 2I, 18 – 18I × 2I,
5 – 5I × I, 7 – 7I × I, 18 – 18I × I}

= {10I – 10I, 14I – 14I, 36I – 36I, 5I – 5I,
7I – 7I, 18I – 18I}

= {0}.

Thus we can have infinite number of subset zero divisors.

If in the above subset semiring if we take

$A = \{t_1 - t_1I, t_2 - t_2I, \ldots, t_n - t_nI\}$ and $B = \{s_1I, s_2I, \ldots, s_mI\}$ (m and n are integers) in S we get $A \times B = \{0\}$.

Hence our claim as m and n are arbitrary and $m, n \in Z^+$.

If we replace Q in example 2.7 by R or Z the result is true.

Thus we have infinite subset semiring which has infinite number of subset zero divisors.

Example 2.8: Let
S = {Collection of all subsets from the complex field C} be the subset semiring. We see S is of infinite order S and has no subset zero divisors.

However S has infinite number of subset units of the form if $A = \{a\}$, $a \in C \setminus \{0\}$ we have a unique $b \in C \setminus \{0\}$ such that if $B = \{b\}$ then $A \times B = \{1\}$.

S is a subset semiring has no subset zero divisors but has infinite number of subset units.

Example 2.9: Let S = {Collection of all subsets from the ring $\langle R \cup I\rangle(g)$ where $g^2 = 0$} be the subset semiring. S has subset zero divisors.

Let $A = \{8g, 5g, \sqrt{3}g, 10g\}$ and
$B = \{0, 2g, -10g, \sqrt{5}g, \sqrt{26}g\} \in S$.

We see

$$\begin{aligned} A \times B &= \{8g, 5g, \sqrt{3}g, 10g\} \times \{0, 2g, -10g, \sqrt{5}g, \sqrt{26}g\} \\ &= \{0\} \text{ as } g^2 = 0. \end{aligned}$$

Thus S has infinite number of subset zero divisors.

Also if A = $\{n - nI \mid n \in Z^+ \cup \{0\}\}$ and B = $\{mI \mid m \in Z^+\}$ then A × B = {0}. So S has infinite number of subset zero divisors.

We also see S has infinite number of subset units. S is an infinite subset semiring which is commutative but has infinite number of subset units and subset zero divisors.

Example 2.10: Let S = {Collection of all subsets from the ring $Z_{11}(g)$ with $g^2 = \{0\}$} be the subset semiring. S has both subset zero divisors and subset units. S is of finite order.

Let A ={10} ∈ S.
A × A = {1}. Let A = {4} and B = {3} ∈ S.
A × B = {4} × {3} = {12} = {1} is a subset unit of S.

Let A = {6} and B = {2} ∈ S.
A × B = {6} × {2} = {6 × 2} = {1} ∈ S.

Thus S has subset units.

Let A = {3g, 5g} and B = {2g, g, 10g} ∈ S.

We see

A × B = {3g, 5g} × {2g, g, 10g} = {0}.

Thus S has subset zero divisors.

It is to be observed Z_{11} is a field, only $Z_{11}(g)$ is the finite dual number. Infact S is also known as the subset semiring of finite dual numbers.

Example 2.11: Let S = {Collection of all subsets from the mixed dual ring $Z_{10}(g_1, g_2)$ where $g_1^2 = 0$ and $g_2^2 = g_2$ with $g_1g_2 = g_2g_1 = 0$} be the subset semiring of finite order.

S has subset idempotents; for take A = $\{5g_2\} \in$ S.

We see

$$\begin{aligned} A \times A &= \{5g_2\} \times \{5g_2\} = \{25\, g_2^2\} \\ &= \{5g_2\} \text{ (as } g_2^2 = g_2\}. \end{aligned}$$

Also A = $\{g_2\} \in$ S is such that

$$\begin{aligned} A \times A &= \{g_2\} \times \{g_2\} = \{g_2^2\} \\ &= \{g_2\} = A \text{ is a subset idempotent of S.} \end{aligned}$$

Take A = $\{6g_2, 5g_2\} \in$ S.

We find

$$\begin{aligned} A \times A &= \{6g_2, 5g_2\} \in \{6g_2, 5g_2\} \\ &= \{(6 \times 6)\, g_2^2, (5 \times 5)\, g_2^2, (6 \times 5)\, g_2^2, \\ &\quad (5 \times 6)\, g_2^2\} \\ &= \{6g_2, 5g_2\} = A \in S. \end{aligned}$$

Thus S has several subset idempotents.

We take M = $\{5g_1, 2g_1, 8g_1, 9g_1, 6g_1\}$ and
N = $\{2g_2, 6g_2, 4g_2\} \in$ S.

We find

$$\begin{aligned} M \times N &= \{5g_1, 2g_1, 8g_1, 9g_1, 6g_1\} \times \{2g_2, 6g_2, 4g_2\} \\ &= \{5g_1 \times 2g_2, 2g_1 \times 2g_2, 8g_1 \times 2g_2, 9g_1 \times 2g_2, \\ &\quad 6g_1 \times 2g_2, 5g_1 \times 6g_2, 2g_1 \times 6g_2, 8g_1 \times 6g_2, \\ &\quad 9g_1 \times 6g_2, 6g_1 \times 6g_2, 5g_1 \times 4g_2, 2g_1 \times 4g_2, \\ &\quad 8g_1 \times 4g_2, 9g_1 \times 4g_2, 6g_1 \times 4g_2\} \\ &= \{0\} \text{ as } g_1 \times g_2 = 0. \end{aligned}$$

Thus S has several subset zero divisors.

We have subset units also given by A = {3} and B = {7} $\in$ S; such that A $\times$ B = {3} $\times$ {7} = {21} = {1} is a subset unit.

A_1 = {9} $\in$ S is such that A_1^2 = {9} $\times$ {9} = {1} is again a subset unit of S.

Example 2.12: Let S = {Collection of all subsets from the ring Z_{12} (g_1, g_2, g_3); $g_1^2 = 0$ and $g_2^2 = g_2$, $g_3^2 = -g_3$, $g_ig_j = g_jg_i = 0$ for $i \neq j$, $1 \leq i, j \leq 3$} be the subset semiring of special mixed dual numbers of finite order.

S has subset units, subset idempotents and subset zero divisors.

Example 2.13: Let S = {Collection of all subsets from the ring Z[x]} be the subset semiring.

S has no subset units or subset idempotents or subset zero divisors.

Infact S is of infinite order and S is also known as the subset semiring of polynomials.

Let

$A = \{5x + 3, 8x^2 + 1, 2x^3 - 1\}$ and
$B = \{x^2 + 3, x^2 - 1, 2x^5 + 4\} \in S$.

We find

$$A + B = \{5x + 3, 8x^2 + 1, 2x^3 - 1\} + \{x^2 + 3, x^2 - 1, 2x^5 + 4\}$$

$$\begin{aligned}
&= \{5x + 3 + x^2 + 3,\ 8x^2 + 1 + x^2 + 3,,\ 2x^3 - 1 + \\
&\quad x^2 + 3,\ 5x + 3 + x^2 - 1,\ 8x^2 + 1 + x^2 - 1,\ 2x^3 - 1 \\
&\quad + x^2 - 1,\ 5x + 3 + 2x^5 + 4,\ 8x^2 + 1 + 2x^5 + 4, \\
&\quad 2x^3 - 1 + 2x^5 + 4\} \\
&= \{x^2 + 5x + 6,\ 9x^2 + 4,\ 2x^3 + x^2 + 2,\ x^2 + 5x + 2, \\
&\quad 9x^2,\ 2x^3 + x^2 - 2,\ 2x^5 + 5x + 7,\ 2x^5 + 8x^2 + 5, \\
&\quad 2x^5 + 2x^3 + 3\} \in S.
\end{aligned}$$

This is the way addition '+' is performed on S.

We find

$$\begin{aligned}
A \times B &= \{5x + 3,\ 8x^2 + 1,\ 2x^3 - 1\} \times \{x^2 + 3,\ x^2 - 1, \\
&\quad 2x^5 + 4\} \\
&= \{5x + 3 \times x^2 + 3,\ 5x + 3 \times x^2 - 1,\ 5x + 3 \times \\
&\quad 2x^5 + 4,\ 8x^2 + 1 \times x^2 + 3,\ 8x^2 + 1 \times x^2 - 1, \\
&\quad 8x^2 + 1 \times 2x^5 + 4,\ 2x^3 - 1 \times x^2 + 3, \\
&\quad 2x^3 - 1 \times x^2 - 1,\ 2x^3 - 1 \times 2x^5 + 4\} \\
&= \{5x^3 + 3x^2 + 9 = 15x,\ 5x^3 + 3x^2 - 5x - 3,\ 10x^6, \\
&\quad 6x^5 + 20x + 12,\ 8x^4 + 25x^2 + 3,\ 9x^4 - 1, \\
&\quad - 7x^2,\ 16x^7 + 2x^5 + 32x^2 + 4,\ 2x^5 - x^2 - 3 + 6x^3, \\
&\quad 2x^5 + 1 - x^2 - 2x^3,\ 4x^8 - 4 + 8x^3 - 2x^5\} \in S.
\end{aligned}$$

This is the way operation × on S is performed.

Thus (S, +, ×) is only an infinite polynomial subset semiring which has no subset zero divisors or subset idempotents or subset units.

We see S is not a strict semiring for we see if

$A = \{5x^3 - 2x^2 + 8x - 7\}$ and
$B = \{-5x^3 + 2x^2 - 8x + 7\}$ are in S, we have

$$\begin{aligned} A + B &= \{5x^3 - 2x^2 + 8x - 7\} + \{-5x^3 + 2x^2 - 8x + 7\} \\ &= \{5x^3 - 2x^2 + 8x - 7 + (-5x^3 + 2x^2 - 8x + 7) \\ &= \{0\} \in S. \end{aligned}$$

Thus S is not a strict subset semiring for we have $A + B = \{0\}$ without A and B being zero in S.

Example 2.14: Let S = {Collection of all subsets from the neutrosophic polynomial ring $\langle R \cup I\rangle[x]$} be the subset polynomial neutrosophic semiring.

We see S has subset zero divisors only of the form $A \times B = \{0\}$ where $A = \{n - nI \mid n \in R^+\}$ and $B = \{mI \mid m \in R\}$ in S are such that $A \times B = \{0\}$.

Thus this subset polynomial neutrosophic semiring has subset zero divisors.

Now we give examples of infinite subset polynomial semirings using the polynomial ring F[x] where $|F| < \infty$.

Example 2.15: Let S = {Collection of all subsets from the polynomial ring $Z_6[x]$} be the subset polynomial semiring of $Z_6[x]$ of infinite order.

S has subset zero divisors but has atleast six subset idempotents of the form

$A = \{3\}$, $B = \{4\}$, $D = \{1, 3\}$, $E = \{1, 4\}$ and $F = \{1, 3, 4\}$, $C = \{3, 4\} \in S$; such that

$$\begin{aligned} A \times A &= \{3\} \times \{3\} \\ &= \{3\} \\ &= A. \end{aligned}$$

$$\begin{aligned} B \times B &= \{4\} \times \{4\} = \{4 \times 4\} \\ &= \{16\} = \{4\} = B. \end{aligned}$$

$$\begin{aligned}C \times C &= \{3, 4\} \times \{3, 4\}\\ &= \{3 \times 3, 3 \times 4, 4 \times 4, 4 \times 3\}\\ &= \{3, 4\} = C.\end{aligned}$$

$$\begin{aligned}E \times E &= \{1, 4\} \times \{1, 4\}\\ &= \{1 \times 4, 4 \times 4, 4 \times 1, 1 \times 1\}\\ &= \{4, 1\} = E.\end{aligned}$$

$$\begin{aligned}D \times D &= \{1, 3\} \times \{1, 3\}\\ &= \{1 \times 1, 1 \times 3, 3 \times 1, 3 \times 3\}\\ &= \{1, 3\} = D \text{ and}\end{aligned}$$

$$\begin{aligned}F \times F &= \{1, 3, 4\} \times \{1, 3, 4\}\\ &= \{1 \times 1, 3 \times 1, 4 \times 1, 1 \times 3, 3 \times 3, 4 \times 3,\\ &\quad 1 \times 4, 3 \times 4, 4 \times 4\}\\ &= \{1, 4, 3\} = F, \text{ are subset idempotents of S.}\end{aligned}$$

Now consider $A = \{2x^2 + 4x + 2, 4x^3 + 2x + 4\}$ and $B = \{3x^3 + 3x^8 + 3x + 3\} \in S$.

We see

$$\begin{aligned}A \times B \;&=\; \{2x^2 + 4x + 2, 4x^3 + 2x + 4\} \times \{3x^3 + 3,\\ &\qquad 3x^8 + 3x + 2\}\\ &=\; \{(2x^2 + 4x + 2) \times 3x^3 + 3, 4x^3 + 2x + 4 \times\\ &\qquad 3x^3 + 3, 2x^2 + 4x + 2 \times 3x^8 + 3x + 3,\\ &\qquad 4x^3 + 2x + 4 \times 3x^8 + 3x + 3\}\\ &=\; \{0\}.\end{aligned}$$

Thus S has subset zero divisors.

So we will have problems while defining subset degree of these subset polynomials; we have infinite number of subset zero divisors in S.

Example 2.16: Let S = {Collection of all subsets from the polynomial ring $R = [Z_{12}(g)][x]$; $g^2 = 0$} be the dual number coefficient polynomial subset semiring of infinite order of the ring R. S has subset zero divisors and subset idempotents.

Example 2.17: Let S = {Collection of all subsets from the polynomial ring $(C(\langle Z_{24} \cup I\rangle (g_1, g_2, g_3))[x]\}$ be the subset polynomial complex finite neutrosophic modulo integer semiring neutrosophic special mixed dual numbers.

We see S has subset zero divisors, subset idempotents and subset units.

Example 2.18: Let S = {Collection of all subsets from the ring $R = Z_{12}(g) \times Z_{15}(g_1)$ where $g^2 = 0$ and $g_1^2 = g_1$} be the subset semiring of the ring R. S has subset units, subset zero divisors and subset idempotents.

Now having seen several examples of subset semirings; we proceed onto study and describe their substructures and other properties by examples.

Example 2.19: Let
S = {Collection of all subsets from the ring Z_{12}} be the subset semiring of Z_{12}. Clearly S has subset subsemirings and subset semiring ideals.

M_1 = {Collection of all subsets from the subring $P_1 = \{0, 2, 4, 6, 8, 10\} \subseteq Z_{12}\} \subseteq S$ is a subset subsemiring which is also a subset semiring ideal of S.

For if $A = \{2, 0, 6\} \in M_1$ and $B = \{3, 1, 5\} \in S$ then

$$\begin{aligned} A \times B &= \{2, 0, 6\} \times \{3, 1, 5\} \\ &= \{0 \times 3, 2 \times 3, 6 \times 3, 0 \times 1, 2 \times 1, 6 \times 1, 0 \times 5, \\ &\quad 2 \times 5, 6 \times 5\} \\ &= \{0, 6, 2, 4\} \in M_1. \end{aligned}$$

It is easily verified M_1 is a subset semiring ideal of the subset semiring.

Consider M_2 = {Collection of all subsets from the subring $P_2 = \{0, 3, 6, 9\} \subseteq Z_{12}\} \subseteq$ S be the subset subsemiring of S as well as subset semiring ideal of the subset semiring S.

M_3 = {Collection of all subsets from the subring $P_3 = \{0, 6\} \subseteq Z_{12}\} \subseteq$ S be the subset subsemiring of S and M_3 is also a subset semiring ideal of S.

Now $N_1 = \{\{1\}, \{0\}, \{2\}, \{3\}, \{4\}, \ldots, \{10\}, \{11\}\} \subseteq S$. N_1 is a subset subsemiring of S but is not a subset semiring ideal of S.

For $\{1\} \in N_1$, so if $A = \{2, 4, 6, 3\} \in S$ we see

$$\begin{aligned} A \times \{1\} &= \{2, 4, 6, 3\} \times \{1\} \\ &= \{2 \times 1, 4 \times 1, 6 \times 1, 3 \times 1\} \\ &= \{2, 4, 6, 3\} \notin N_1. \end{aligned}$$

Infact if $N_2 = \{\{0\}, \{2\}, \{4\}, \{6\}, \{8\}, \{10\}\} \subseteq S$; N_2 is only a subset subsemiring of S but is not a subset semiring ideal of S.

For if we take $A = \{1, 2, 3, 4, 7, 10\} \in S$ we consider $A \times B$ where $B = \{2\} \in N$.

$$\begin{aligned} A \times B &= \{1, 2, 3, 4, 7, 10\} \times \{2\} \\ &= \{1 \times 2, 2 \times 2, 3 \times 2, 4 \times 2, 7 \times 2, 10 \times 2\} \\ &= \{2, 4, 6, 8\} \notin N_2, \text{ hence the claim.} \end{aligned}$$

Thus we have shown by this example that S has subset subsemirings which are not subset semiring ideals of S.

Example 2.20: Let
S = {Collection of all subsets from the field Z_7} be the subset semiring.

Consider $P = \{\{1, 2, 3, 4, 5, 6, 0\}, \{0\}\} \subseteq S$; P is a subset subsemiring as well as subset semiring ideal of S.

For take $A = \{0, 2, 4\} \in S$.

$$\begin{aligned}
A \times B &= \{2, 0, 4\} \times \{0, 1, 2, 3, 4, 5, 6\} \\
&\qquad (\text{where } B = \{0, 1, 2, 3, 4, 5, 6\}) \\
&= \{2 \times 0, 2 \times 1, 2 \times 2, 2 \times 3, 2 \times 4, 2 \times 5, 2 \times 6, \\
&\quad 0 \times 1, 0 \times 2, 0 \times 3, 0 \times 4, 0 \times 5, 0 \times 6, 4 \times 1, \\
&\quad 4 \times 2, 4 \times 3, 4 \times 4, 4 \times 5, 4 \times 6\} \\
&= \{0, 2, 4, 6, 1, 3, 5\} \in P;
\end{aligned}$$

and $\{0\} \times A = \{0\}$ for all $A \in S$.

Thus $P = \{\{0\}, \{1, 2, 3, 4, 5, 6, 0\}\}$ is a subset semiring ideal of S.

We have seen subset subsemirings which are subset semiring ideals and those which are just only subset subsemirings.

In view of this we have the following theorem.

THEOREM 2.1: *Let S be a subset semiring of a ring. Every subset ideal of a subset semiring is a subset subsemiring of S. However all subset subsemirings of S need not in general be a subset semiring ideal of S.*

The proof is direct and hence left as an exercise to the reader.

It is just important to recall every ring is trivially a semiring but a semiring in general is not a ring. So we can define a subset semiring to be a super Smarandache subset semiring if it contains a subset ring.

We will give examples of them before we proceed to define more properties.

Example 2.21: Let
S = {Collection of all subsets from the ring Z_6} be the subset semiring. S is a super Smarandache subset semiring as A = {{0}, {1}, {2}, {3}, {4}, {5}} $\subseteq$ S is a subset ring of S, hence the claim.

Example 2.22: Let
S = {Collection of all subsets from the ring $C(Z_5)$} be the super Smarandache subset semiring of the ring. Infact S has two subset rings M_1 = {{0}, {1}, {2}, {3}, {4}} $\subseteq$ S and M_2 = {{a} | a $\in$ $C(Z_5)$} $\subseteq$ S are subset rings. Hence the claim.

Thus we see subset semiring constructed using subsets of a ring are defined as super Smarandache subset semiring.

Now we give some more examples of substructures in subset semirings of type I.

Example 2.23: Let
S ={Collection of all subsets from the ring Z} be the subset semiring of the ring Z. P_2 = {Collection of all subsets from the subring 2Z = {0 ± 2, …, ± 2n …}} $\subseteq$ S is a subset subsemiring and also a subset semiring ideal of the semiring S.

Infact S has infinite number of subset subsemirings which are subset semiring ideals.

For P_n = {Collection of all subsets from the subring nZ = {0, ± n, ± 2n, …} $\subseteq$ Z} $\subseteq$ S is a subset subsemiring which is a subset semiring ideal of S for every n; n $\in$ $Z^+ \setminus \{1\}$.

Example 2.24: Let
S = {Collection of all subsets from the ring R the field of reals} be the subset semiring of R. R has infinite number of subset subsemirings.

Take P_n = {Collection of all subsets from the subring $nZ \subseteq R$, $n Z \subseteq R$, $n \in Z^+$} $\subseteq$ S be the subset subsemiring of S. P_n is not a subset semiring ideal of S.

P_Q = {Collection of all subsets from the subring $Q \subseteq R$} $\subseteq$ S is the subset subsemiring of S. P_Q is a not a subset semiring ideal only a subset subsemiring of S.

Thus S has infinite number of subset subsemirings which are not subset semiring ideals.

Take M = {{a} | a $\in$ R} {0}}; the collection of subset singleton sets from R, denote by M_1.

M = {M_1, {0}} $\subseteq$ S is a subset subsemiring which is also a subset semiring ideal of S.

Example 2.25: Let
S = {Collection of all subsets from the ring $\langle R \cup I\rangle$} be the subset neutrosophic semiring.

S has subset subsemirings which are not subset semiring ideals of S for take P = {All subsets from the ring $\langle Z \cup I\rangle$} $\subseteq$ S, a subset subsemiring which is not a subset semiring ideal of S.

L = {All subsets from the ring Z} $\subseteq$ P $\subseteq$ S is a subset subsemiring of S which is not a subset semiring ideal of S.

Let
T = {Collection of all subsets from the ring $\langle Q \cup I\rangle$} $\subseteq$ S be the subset subsemiring which is not a subset semiring ideal of S.

Take
V = {Collection of all subsets from the ring $Q \subseteq \langle R \cup I\rangle$} $\subseteq$ S be the subset subsemiring of S which is not a subset semiring ideal of S.

Example 2.26: Let
S = {Collection of all subsets from the ring $\langle C \cup I\rangle$} be the subset semiring. S has many subset subsemirings which are not subset semiring ideals.

Take
L = {Collection of all subsets from the subring $Z \subseteq \langle C \cup I\rangle$} be the subset subsemiring of S which is not a subset semiring ideal of S.

T = {Collection of all subsets of the subring $\langle Z \cup I\rangle \subseteq \langle C \cup I\rangle$} $\subseteq$ S is again a subset subsemiring of S which is not a subset semiring ideal of S.

W = {Collection of all subsets from the subring $C \subseteq \langle C \cup I\rangle$} $\subseteq$ S be the subset subsemiring of S which is not a subset semiring ideal of S.

We can have infinite number of subset subsemirings which are not subset semiring ideals of S.

Example 2.27: Let
S = {Collection of all subsets from the ring Z_{15}} be the subset semiring of the ring Z_{15}. Take P_1 = {Collection of all subsets from the subring $M = \{0, 5, 10\} \subseteq Z_{15}$} $\subseteq$ S be the subset subsemiring which is also a subset semiring ideal of S.

P_2 = {Collection of all subsets from the subring $N = \{0, 3, 6, 9, 12\} \subseteq Z_{15}$} $\subseteq$ S be the subset subsemiring of S. P_2 is also subset semiring ideal of S.

Let $T_1 = \{\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}, \ldots, \{14\}\} \subseteq S$ is a subset subsemiring of S which is not a subset semiring ideal of S.

$T_2 = \{\{0\}, \{5\}, \{10\}\} \subseteq S$ is a subset subsemiring which is not a subset semiring ideal of S.

$T_3 = \{\{0\}, \{3\}, \{6\}, \{9\}, \{12\}\} \subseteq S$ is subset subsemiring of S which is not a subset semiring ideal of S.

Infact S has both subset subsemirings which are not subset semiring ideals as well as S has subset semiring ideals.

Example 2.28: Let
S = {Collection of all subsets of the ring Z_{24}} be the subset semiring. P_1 = {Collection of all subsets from the subring $M_1 = \{0, 2, 4, 6, 8, \ldots, 22\} \subseteq Z_{24}\} \subseteq S$ is the subset subsemiring which is a subset semiring ideal of S.

P_2 = {Collection of all subsets of the subring $M_2 = \{0, 4, 8, 12, 16, 20\} \subseteq Z_{24}\} \subseteq S$ is again a subset subsemiring which is also a subset semiring ideal of S.

P_3 = {Collection of all subsets of the subring $M_3 = \{0, 8, 16\} \subseteq Z_{24}\} \subseteq S$ is again a subset subsemiring which a subset semiring ideal of S.

P_4 = {Collection of all subsets of the subring $M_4 = \{0, 1, 2\} \subseteq Z_{24}\} \subseteq S$ is again subset subsemiring which is also a subset semiring ideal of S.

P_5 = {Collection of all subsets of the subring $M_5 = \{0, 6, 12, 18\} \subseteq Z_{24}\} \subseteq S$ is again a subset subsemiring which is also a subset semiring ideal of S.

P_6 = {Collection of all subsets of the subring $M_6 = \{0, 3, 6, 9, 12, 15, 18, 21\} \subseteq Z_{24}\} \subseteq S$ is again a subset subsemiring which is also a subset semiring ideal of S.

We have 6 subset subsemirings of S which are subset semiring ideals of S.

Related with these six subset subsemirings $P_1, P_2, \ldots, P_6$, we construct $V_1, V_2, \ldots, V_6$ where $V_1 = \{\{0\}, \{2\}, \{4\}, \{6\}, \ldots, \{22\}\} \subseteq S$ is a subset subsemiring of S. This is not a subset semiring ideal of S.

Take A = {3, 5, 7, 1, 6} $\in$ S. A × {4} = {12, 20, 28, 4, 24} $\notin$ V_1 so V_1 is not a subset semiring ideal of S.

If we take V_2 = {{0}, {4}, {8}, {12}, {16}, {20}} $\subseteq$ S, we see V_2 is only a subset subsemiring and is not a subset semiring ideal of S.

V_3 = {{0}, {8}, {16}} $\subseteq$ S where V_3 is only a subset subsemiring and is not a subset semiring ideal of S.

Thus S has subset semiring ideal and subset subsemirings which are not subset semiring ideals of S.

We will give one more examples before we enunciate a result.

Example 2.29: Let
S = {Collection of all subsets from the ring Z_{20}} be the subset semiring of Z_{20}.

Consider P_1 = {Collection of all subsets from the subring M_1 = {0, 2, 4, 6, …, 18} $\subseteq$ Z_{20}} $\subseteq$ S; P_1 is a subset subsemiring as well as subset semiring ideal of S.

P_2 = {Collection of all subsets of the subring M_2 = {0, 4, 8, 12, 16}} $\subseteq$ S, P_2 is a subset subsemiring which is a subset semiring ideal of S.

P_3 = {Collection of all subsets from the subring M_3 = {0, 5, 10, 15} $\subseteq$ Z_{20}} $\subseteq$ S is again a subset subsemiring which is a subset semiring ideal of S.

P_4 = {Collection of all subsets form the subring M_4 = {0, 10} $\subseteq$ Z_{20}} $\subseteq$ S is again a subset subsemiring of S which is also a subset semiring ideal of S.

Also N_1 = {{0}, {10}} $\subseteq$ S is only subset subsemiring which is not a subset semiring ideal of S.

N_2 = {{0}, {5}, {10}, {15}} $\subseteq$ S is again a subset subsemiring which is not a subset semiring ideal of S.

W_3 = {{0}, {4}, {8}, {12}, {16}} $\subseteq$ S is also a subset subsemiring which is not a subset semiring ideal of S.

Thus in view of all these we have the following theorem.

THEOREM 4.2: *Let*
S = {Collection of all subsets from the ring R} be the subset semiring of type I.

(i) S has atleast as many subset subsemirings (ideals) as subrings (ideals) of R.

(ii) S has atleast same number of subset subsemirings which are not subset semiring ideals as mentioned in (i).

Proof: If R the ring over which the subset semiring S is built and if R has n ideals then we see S has n number of subset semiring ideals for if I is an ideal of R take

P = {Collection of all subsets from I} $\subseteq$ S is again a subset subsemiring of S which is also a subset semiring ideal of S.

If we take I = {0, a_1, …, a_m} $\subseteq$ R is the elements of I then take V = {{0}, {a_1}, …, {a_m}} $\subseteq$ S. It is easily verified V is a subset subsemiring of S but is not a subset semiring ideal of S for if we take A = {s_1, …, s_t | $s_i \in$ R} then A × {a_s} $\neq$ {a_i} in general for any i as A × {a_s} has more elements in general. So V can only be a subset subsemiring of S.

Hence the claim.

Example 2.30: Let
S = {Collection of all subsets from the ring Z_6} be the subset semiring of type I. P_1 = {Collection of all subsets from the

subring / ideal $M_1 = \{0, 3\} \subseteq Z_6\} \subseteq S$ is again a subset subsemiring which is also a subset semiring ideal of S.

P_2 = {Collection of all subsets from the subring / ideal $M_2 = \{0, 2, 4\} \subseteq Z_6\} \subseteq S$ is again a subset subsemiring as well as subset semiring ideal of S.

Take $N_1 = \{\{0\}, \{3\}\} \subseteq S$, N_1 is only a subset subsemiring and is not a subset semiring ideal of S for if $A = \{1, 2\} \in S$

$$\begin{aligned} A \times \{3\} &= \{1, 2\} \times \{3\} \\ &= \{3, 6\} \\ &= \{0, 3\} \notin N_1. \end{aligned}$$

Hence the claim.

Take $N_2 = \{\{0\}, \{2\}, \{4\}\} \subseteq S$; N_2 is only a subset subsemiring and not a subset semiring ideal of S.

$$\begin{aligned} &\text{For if } A = \{1, 5\} \in S. \\ A \times \{4\} &= \{1, 5\} \times 4 \\ &= \{4, 20\} \\ &= \{4, 2\} \notin N_2. \end{aligned}$$

So N_2 is only a subset subsemiring and not a subset semiring ideal of S.

$N_3 = \{\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}\} \subseteq S$. N_3 is only a subset subsemiring of S and not a semiring ideal of S.

Inview of all these we have the following theorem.

THEOREM 2.3: *Let*
S = {Collection of all subsets of a ring R} be the subset semiring of the ring R. S has subset subsemirings which are not subset semiring ideals of S.

Proof: Follows from the simple fact if M = {{a} | a $\in$ R} (Collection of all singleton sets of R) then M $\subseteq$ S is only a subset subsemiring which is not a subset semiring ideal of S.

Now we proceed onto study more about non commutative rings and their related subset semirings which are non commutative.

Example 2.31: Let
S = {Collection of all subsets from the group ring Z_2S_3} be the subset semiring.

S is a subset semiring which is non commutative.

However S has subset subsemirings which are commutative.

P_1 = {Collection of all subsets from the groupring Z_2T_1 where

$$T_1 = \left\{\begin{pmatrix}1 & 2 & 3\\1 & 2 & 3\end{pmatrix},\begin{pmatrix}1 & 2 & 3\\2 & 3 & 1\end{pmatrix},\begin{pmatrix}1 & 2 & 3\\3 & 1 & 2\end{pmatrix}\right\}\right\} \subseteq S$$

is a subset subsemiring of S which is commutative subset subsemiring.

P_2 = {Collection of all subsets from the group ring Z_2T_2 where

$$T_2 = \left\{\begin{pmatrix}1 & 2 & 3\\1 & 2 & 3\end{pmatrix},\begin{pmatrix}1 & 2 & 3\\1 & 3 & 2\end{pmatrix}\right\}\right\} \subseteq S$$

is again a subset subsemiring of S which is a commutative subset subsemiring of S.

Take $A = \left\{\begin{pmatrix}1 & 2 & 3\\2 & 3 & 1\end{pmatrix},\begin{pmatrix}1 & 2 & 3\\1 & 3 & 2\end{pmatrix}\right\}$ and

$$B = \{1 + \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}\} \in S.$$

We find

$$A \times B = \left\{\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}\right\} \times \{1 + \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} +$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}\}$$

$$= \{\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \times \Big(1 + \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} +$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}\Big),$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \times \Big(1 + \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}\Big) +$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}\Big)\}$$

$$= \{\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} +$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} + \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix},$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} +$$

$$\left.\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}+\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}\right\} \quad \ldots \text{ I}$$

We now find

$$\text{B} \times \text{A} = \left\{1+\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}+\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}+\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}\right\} \times$$

$$\left\{\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix},\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}\right\}$$

$$= \left\{\left(1+\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}+\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}+\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}\right) \times\right.$$

$$\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix},$$

$$\left(1+\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}+\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}+\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}\right) \times$$

$$\left.\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}\right\}$$

$$= \left\{\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}+\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}+\begin{pmatrix}1&2&3\\1&2&3\end{pmatrix}\right.$$

$$+\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix},\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}+\begin{pmatrix}1&2&3\\1&2&3\end{pmatrix}+$$

$$\left.\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}+\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}\right\} \quad \ldots \text{ II}$$

Clearly I and II are distinct so A × B ≠ B × A; hence S is a non commutative subset semiring of finite order.

Example 2.32: Let
S = {Collection of all subsets from the group ring ZS_3} be subset semiring of ZS_3, S is a non commutative subset semiring of infinite order.

S has several infinite order subset subsemirings both commutative and non commutative.

However S has subset zero divisors for take $A = \{1 - g_1\}$ and $B = \{1 + g_1\} \in S$.

We see $A \times B = \{1 - g_1\} \times \{1 + g_1\}$

(where $g_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$)

$$= \{(1 - g_1)(1 + g_1)\}$$
$$= \{1 - g_1 + g_1 - g_1^2\}$$
$$= \{1 - 1\} = \{0\}.$$

Take $A = \{1 + g_1 + g_2 + g_3 + g_4 + g_5\}$ and $B = \{1 - g_1\} \in S$,

where $g_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$, $g_2 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$, $g_3 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$,

$g_4 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ and $g_5 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ are in S_3.

$A \times B$
$= \{1 + g_1 + g_2 + g_3 + g_4 + g_5\} \times \{1 - g_1\}$
$= \{1 + g_1 + g_2 + g_3 + g_4 + g_5 \times (1 - g_1)\}$
$= \{1 + g_1 + g_2 + g_3 + g_4 + g_5 - g_1 - 1 - g_4 - g_5 - g_2 - g_3\}$
$= \{0\}$,

hence again a subset zero divisor of S.

Thus non commutative subset semiring S of ZS_3 has subset zero divisors.

However S has no subset idempotents.

Example 2.33: Let
S = {Collection of all subsets from the groupring QS_3} be the subset semiring of the group ring QS_3 which is non commutative and of infinite order.

This has both subset zero divisors as well as subset idempotents.

For take $A = \{\frac{1}{2}(1 + g_1) \mid g_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}\} \in S$.

$$\begin{aligned} A \times A &= \{\frac{1}{2}(1 + g_1)\} \times \{\frac{1}{2}(1 + g_1)\} \\ &= \{\frac{1}{2}(1 + g_1) \times \frac{1}{2}(1 + g_1)\} \\ &= \{\frac{1}{4}(1 + g_1)^2\} \\ &= \{\frac{1}{4}(1 + g_1^2 + 2g_1)\} \\ &= \{\frac{1}{4}(1 + 1 + 2g_1)\} \\ &= \{\frac{1}{2}(1 + g_1)\} = A \in S. \end{aligned}$$

Thus A is a subset idempotent of S.

Also B = $\{\frac{1}{3}(1 + g_4 + g_5)\} \in$ S is such that B × B = B so B is also a subset idempotent of S.

Now C = $\{\frac{1}{6}(1 + g_1 + g_2 + g_3 + g_4 + g_5)\} \in$ S is such that C × C = C so C is also a subset idempotent of S.

Infact S has also subset zero divisors.

Example 2.34: Let S = {Collection of all subsets from the groupring RS_3} be the subset semiring of RS_3. S is non commutative and has both subset zero divisors and subset units.

S has subset subsemirings and which are not subset semiring ideals.

For consider P = {Collection of all subsets from the subring QS_3} $\subseteq$ S; P is a subset subsemiring which is not a subset semiring ideal of S.

Infact S has infinite number of subset subsemirings which are not subset semiring ideals of S.

Example 2.35: Let S = {Collection of all subsets from M = {3 × 3 matrices with entries from Z_{12}}} be the subset semiring of the ring M.

Clearly S is a non commutative matrix subset semiring of finite order. S has both subset zero divisors, subset idempotents and subset subsemirings.

Example 2.36: Let S = {Collection of all subsets from the matrix ring M = $\{(a_1, a_2, a_3, a_4) \mid a_i \in Z_{15}, 1 \leq i \leq 4\}$} be the subset semiring of Z_{15} of finite order which is commutative.

S has subset zero divisors and subset idempotents.

Take A = {(0, 2, 4, 5), (0, 0, 7, 2), (0, 1, 2, 0), (0, 5, 0, 1)} and B = {(7, 0, 0, 0), (8, 0, 0, 0), (5, 0, 0, 0) (1, 0, 0, 0)} be in S.

We see

$$\begin{aligned} A \times B &= \{(0, 2, 4, 5), (0, 0, 7, 2), (0, 1, 2, 0), (0, 5, 0, 1)\} \times \\ &\quad \{(7, 0, 0, 0), (8, 0, 0, 0), (5, 0, 0, 0), (1, 0, 0, 0)\} \\ &= \{(0, 2, 4, 5) \times (7, 0, 0, 0), (0, 2, 4, 5) \times (8, 0, 0, 0), \\ &\quad (0, 1, 2, 0) \times (7, 0, 0, 0), (0, 2, 4, 5) \times (5, 0, 0, 0), \\ &\quad (0, 2, 4, 5) \times (1, 0, 0, 0), (0, 1, 2, 0) \times (8, 0, 0, 0), \\ &\quad (0, 1, 2, 0) \times (5, 0, 0, 0), (0, 1, 2, 0) \times (1, 0, 0, 0), \\ &\quad (0, 0, 7, 2) \times (7, 0, 0, 0), (0, 0, 7, 2) \times (8, 0, 0, 0), \\ &\quad (0, 0, 7, 2) \times (5, 0, 0, 0), (0, 0, 7, 2) \times (1, 0, 0, 0), \\ &\quad (0, 5, 0, 1) \times (8, 0, 0, 0), (0, 5, 0, 1) \times (7, 0, 0, 0), \\ &\quad (0, 5, 0, 1) \times (1, 0, 0, 0), (7, 0, 0, 0) \times (5, 0, 0, 0)\} \\ &= \{(0, 0, 0, 0)\} \in S. \end{aligned}$$

Thus S has several subset zero divisors.

S has subset idempotents. For take $A_1 = \{(0\ 1\ 0\ 1)\} \in S$.

$A_1 \times A_1 = \{(0\ 1\ 0\ 1)\} = A_1$.

$A_2 = \{(10, 1, 10, 0)\} \in S$ is such that $A_2 \times A_2 = A_2$ and so on.

Example 2.37: Let

$$P = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \middle|\ a_i \in Z_{12}, 1 \leq i \leq 5 \right\}$$

be the ring of column matrices under natural product $\times_n$. P is a commutative finite ring.

Let S = {Collection of all subsets of P} be the subset semiring of the ring P. S has subset idempotents and subset zero divisors.

$$\text{Let } A = \left\{\begin{bmatrix}3\\0\\1\\2\\4\end{bmatrix}, \begin{bmatrix}5\\1\\2\\5\\7\end{bmatrix}\right\} \text{ and } B = \left\{\begin{bmatrix}2\\0\\1\\3\\7\end{bmatrix}, \begin{bmatrix}4\\5\\2\\3\\0\end{bmatrix}\right\} \in S.$$

$$\text{We find } A + B = \left\{\begin{bmatrix}3\\0\\1\\2\\4\end{bmatrix}, \begin{bmatrix}5\\1\\2\\5\\7\end{bmatrix}\right\} + \left\{\begin{bmatrix}2\\0\\1\\3\\7\end{bmatrix}, \begin{bmatrix}4\\5\\2\\3\\0\end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix}3\\0\\1\\2\\4\end{bmatrix} + \begin{bmatrix}2\\0\\1\\3\\7\end{bmatrix}, \begin{bmatrix}3\\0\\1\\2\\4\end{bmatrix} + \begin{bmatrix}4\\5\\2\\3\\0\end{bmatrix} \begin{bmatrix}5\\1\\2\\5\\7\end{bmatrix} + \begin{bmatrix}2\\0\\1\\3\\7\end{bmatrix}, \begin{bmatrix}5\\1\\2\\5\\7\end{bmatrix} + \begin{bmatrix}4\\5\\2\\3\\0\end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix}5\\0\\2\\5\\11\end{bmatrix}, \begin{bmatrix}7\\5\\3\\5\\4\end{bmatrix}, \begin{bmatrix}7\\1\\3\\8\\2\end{bmatrix}, \begin{bmatrix}9\\6\\4\\8\\7\end{bmatrix}\right\}.$$

This is the way the operation of + is performed on S.

Now we find

$$A \times B = \left\{ \begin{bmatrix} 3 \\ 0 \\ 1 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ 2 \\ 5 \\ 7 \end{bmatrix} \right\} \times_n \left\{ \begin{bmatrix} 2 \\ 0 \\ 1 \\ 3 \\ 7 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 2 \\ 3 \\ 0 \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} 3 \\ 0 \\ 1 \\ 2 \\ 4 \end{bmatrix} \times_n \begin{bmatrix} 2 \\ 0 \\ 1 \\ 3 \\ 7 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 1 \\ 2 \\ 4 \end{bmatrix} \times_n \begin{bmatrix} 4 \\ 5 \\ 2 \\ 3 \\ 0 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \\ 2 \\ 5 \\ 7 \end{bmatrix} \times_n \begin{bmatrix} 2 \\ 0 \\ 1 \\ 3 \\ 7 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ 2 \\ 5 \\ 7 \end{bmatrix} \times_n \begin{bmatrix} 4 \\ 5 \\ 2 \\ 3 \\ 0 \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} 6 \\ 0 \\ 1 \\ 6 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 2 \\ 6 \\ 0 \end{bmatrix}, \begin{bmatrix} 10 \\ 0 \\ 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 8 \\ 5 \\ 4 \\ 3 \\ 0 \end{bmatrix} \right\} \in S.$$

This is the way natural product is defined on S.

We see S is a finite commutative subset semiring. S has subset zero divisors and subset idempotents.

$$\text{For take } A = \left\{\begin{bmatrix} 0 \\ 6 \\ 0 \\ 6 \\ 6 \end{bmatrix}\right\} \in S.$$

$$\text{We see } A \times_n A = \left\{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\right\}.$$

So A is a subset zero divisor.

$$\text{Let } A = \left\{\begin{bmatrix} 0 \\ 4 \\ 2 \\ 0 \\ 0 \end{bmatrix}\right\} \text{ and } B = \left\{\begin{bmatrix} 6 \\ 6 \\ 6 \\ 3 \\ 3 \end{bmatrix}\right\} \text{ be in S.}$$

$$\text{We see } A \times_n B = \left\{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\right\} \text{ is again a subset zero divisor in S.}$$

Take $M = \left\{\begin{bmatrix}4\\0\\4\\0\\9\end{bmatrix}\right\} \in S$. We see $M \times_n M = \left\{\begin{bmatrix}4\\0\\4\\0\\9\end{bmatrix}\right\} \in S$;

M is a subset idempotent of S. S has subset nilpotent, subset zero divisors and subset idempotents.

Infact S has subset subsemirings and subset semiring ideals.

For take $P_1 = \left\{\begin{bmatrix}a_1\\0\\0\\0\\0\end{bmatrix} \middle| a_i \in Z_{12}\right\}$ be the matrix subring.

Let T_1 = {Collection of all subsets from the matrix subring P_1 of P} $\subseteq$ S, T_1 is a subset subsemiring of S which is also a subset semiring ideal of S.

Let $P_2 = \left\{\begin{bmatrix}0\\a_2\\0\\0\\0\end{bmatrix} \middle| a_i \in Z_{12}\right\}$ be the matrix subring of P.

T_2 = {Collection of all subsets of the matrix subring P_2} $\subseteq$ S is also a subset subsemiring as well as subset semiring ideal of S.

Let M = {Collection of all subsets of the matrix ring

$$P_3 = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \middle| \; a_i \in \{2, 0, 4, 6, 8, 10\} \subseteq Z_{12}, 1 \leq i \leq 5 \right\} \subseteq S$$

is again a subset subsemiring which is also a subset semiring ideal of S.

Example 2.38: Let S = {Collection of all subsets from the matrix ring

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \middle| \; a_i \in Z_{18}, 1 \leq i \leq 16 \right\}\}$$

be the subset semiring of the ring M.

S has subset zero divisors, subset units and subset idempotents.

S is of finite order, commutative under natural product $\times_n$ and non commutative under usual product ×.

We see in both cases it has subset subsemirings and subset semiring ideals.

Take V_1 = {Collection of all subsets from the matrix ring

$$T_1 = \left\{ \begin{bmatrix} a_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \middle| \; a_i \in Z_{18} \right\}\}$$

be the subset subsemiring of the subring T_1.

V_1 is also a subset semiring ideal of S.

Likewise V_i = {Collection of all subsets from

$$T_i = \left\{ \left. \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & a_i & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \right| a_i \in Z_{18} \right\} \subseteq M\} \subseteq S$$

is again both a subset subsemiring as well as subset semiring ideal of S.

We can have several such subset subsemirings and subset semiring ideals.

Example 2.39: Let S = {Collection of all subsets from the matrix ring

$$M = \left\{ \left. \begin{bmatrix} a_1 & a_2 & a_3 \\ a_5 & a_6 & a_7 \\ \vdots & \vdots & \vdots \\ a_{28} & a_{29} & a_{30} \end{bmatrix} \right| a_i \in Z, 1 \leq i \leq 30 \right\}\}$$

be the subset semiring under natural product $\times_n$ of matrices.

S is of infinite order S has subset zero divisors and subset idempotents.

Infact S has infinite number of subset subsemirings and subset semiring ideals.

Example 2.40: Let S = {Collection of all subsets from the matrix ring

$$M = \left\{ \left[\begin{array}{ccc} a_1 & \dots & a_{16} \\ a_{17} & \dots & a_{32} \\ a_{33} & \dots & a_{48} \end{array} \right] \middle| \; a_i \in Z_{420}, 1 \leq i \leq 48 \right\}\}$$

be the subset semiring.

S is of finite order, has subset zero divisors, subset idempotents, subset units, subset subsemirings and subset semiring ideals.

Example 2.41: Let
S = {Collection of all subsets from the semigroup ring $Z_{10}S(3)$} be the subset semiring.

S has subset subsemirings. S has subset semiring ideals and subset subsemirings.

Infact S is of finite order with subset zero divisors, subset units and subset idempotents. S is non commutative.

Example 2.42: Let S = {Collection of all subsets from the semigroup ring ZS(7), S(7) the symmetric semigroup} be the subset semiring.

S is an infinite non commutative subset semiring. S has subset subsemirings, subset semiring ideals, subset zero divisors has subset idempotents also.

For if $A = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \right\} \in S.$

We see

$$A \times A = \left\{\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}\right\} \times \left\{\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}\right\}$$

$$= \left\{\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}\right\} = A.$$

Thus S has subset idempotents.

We can say if the semigroup over which the ring, the semigroup ring is taken is such that the semigroup has non trivial idempotents then the subset semiring has idempotents.

In view of this we have the following theorem.

THEOREM 2.4: *Let S = {Collection of all subsets of the semigroup ring ZP of the semigroup P over the ring of integers Z where P is a semigroup with idempotents} be the subset semiring. Then S has non trivial subset idempotents.*

Note if P is a group then S has no subset idempotents only subset units.

Also if P has no idempotents then S has no subset idempotents.

Finally if P is a semigroup such that for all $a \in P$. $a^2 = a$ and $a.b = 0$ if $a \neq b$ for every $a, b \in P$ then we see S has several subset idempotents which are not singletons.

The proof is direct and hence left as an exercise to the reader.

Example 2.43: Let S = {Collection of all subsets from the semigroup ring ZP where P is a semigroup P = $\{Z_{12}, \times\}$} be the subset semiring.

S has subset idempotents and subset zero divisors.

For take $A_1 = \{4\}$, $A_2 = \{9\} \in S$ we see
$A_1 \times A_1 = \{4\} \times \{4\} = \{4 \times 4\} = \{16\} = \{4\} = A_1$.
$A_2 \times A_2 = \{9\} \times \{9\} = \{9 \times 9\} = \{81\} = \{9\} = A_2$.

Also $A_3 = \{1, 9\} \in S$ is such that
$A_3 \times A_3 = \{1, 9\} \times \{1, 9\} = \{1, 9\} = A_3$.

$A_4 = \{0, 1, 9\} \in S$ is such that
$A_4 \times A_4 = \{0, 1, 9\} \times \{0, 1, 9\} = A_4$.

Thus we A_1, A_2, A_3 and A_4 in S are such that $A_i^2 = A_i$ for $1 \leq i \leq 4$. Hence A_1, A_2, A_3 and A_4 in S are subset idempotents of S.

Take $A_5 = \{0, 1, 9, 4\} \in S$.

$$A_5 \times A_5 = \{0, 1, 9, 4\} \times \{0, 1, 9, 4\}$$
$$= \{0, 1, 9, 4\} = A_5$$

is again a subset idempotent of S.

Let A = {6} and B = $\{4, 2, 8\} \in S$ we see $A \times B = \{0\}$ is a subset zero divisor of S.

Example 2.44: Let
S = {Collection of all subsets from the semigroupring RS(10)} be the subset semiring of infinite order which is non commutative S has subset idempotents.

S has subset zero divisors. S has subset semiring ideals as well as subset subsemirings.

Example 2.45: Let
S = {Collection of all subsets of the ring $Z_5S(3) \times Z_2D_{27}$} be the subset semiring of finite order. S is non commutative and has subset zero divisors and subset units.

For take
A = {(a, 0) | a ∈ $Z_5S(3)$} and B = {(0, b) | b ∈ Z_2D_{27}} ∈ S.

We see
A × B = {(a, 0) × (0, b) | a ∈ $Z_5(S(3))$ and b ∈ Z_2D_{27}} = {(0,0)}.

Hence S has subset zero divisors.

$$\text{Take } A = \left\{\left(\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}, e\right)\right\} \in S.$$

We see A × A = {(1, 1)} is a subset unit of S. So A is a subset unit of S.

Example 2.46: Let
S = {Collection of all subsets from the ring $Z_5 \times Z_{10} \times Z_7$} be the subset semiring. S has subset zero divisors, subset idempotents and so on.

Let
A = {(a, 0, 0) | a ∈ Z_5} and B = {(0, b, c) | b ∈ Z_{10}, c ∈ Z_7} ∈ S.
We see A × B = {(0, 0, 0)} is a subset zero divisor of S.

Let A = {(0, 5, 0)} ∈ S; A × A = {(0, 5, 0)} = A is a subset idempotent of S.

Also B = {(1, 5, 1)} ∈ S is also a subset idempotent of S. D = {(0, 5, 1)} ∈ S is also a subset idempotent of S.

Example 2.47: Let
S = {Collection of all subsets from the ring R = $Z \times Z_{16} \times Z_{15}$} be the subset semiring. S has subset zero divisors, subset

idempotents and subset units. S has also subset subsemirings which are subset semiring ideals.

Take $P_1 = \{(a, 0, 0) \mid a \in Z\} \subseteq S$ we see P_1 is only a subset subsemiring but P_1 is not a subset semiring ideal of S.

For if we take
$A = \{(5, 8, 9), (11, 2, 0), (4, 7, 0), (-11, 8, 0)\}$ in S and
$B = \{(4, 0, 0)\} \in P_1$.

We find

$$A \times B = \{(5, 8, 9), (11, 2, 0), (4, 7, 0), (-11, 8, 0)\} \times \{(4, 0, 0)\}$$

$$= \{(20, 0, 0), (4, 4, 0, 0), (16, 0, 0), (-44, 0, 0)\} \in S$$ but $\{(20, 0, 0), (44, 0, 0), (16, 0, 0), (-44, 0, 0)\} \notin P_1$ hence P_1 is only a subset subsemiring and not a subset semiring ideal of S.

Let $P_2 = \{(0, a, 0) \mid a \in Z_{16}\} \subseteq S$ be the collection of all subset subsemiring of S. P_2 is only subset subsemiring but is not a subset semiring ideal of S.

$P_3 = \{(0, 0, a) \mid a \in Z_{15}\} \subseteq S$ be the subset subsemiring of S. Clearly P_3 is not a subset semiring ideal of S. Thus S has subset subsemirings which are not subset semiring ideals of S.

We now give examples of subsemirings ideals of S.

Take V_1 = {Collection of all subsets from the subring $T_1 = \{Z \times \{0\} \times \{0\}\} \subseteq S$; V is a subset semiring ideal of S.

We see V_n = {Collection of all subsets from the subring $T_2 = \{nZ \times \{0\} \times \{0\}\}\}$ $(2 \leq n < \infty)$ be subset semiring ideals of S.

We have an infinite collection of such subset semiring ideals.

V_3 = {Collection of all subsets from the subring; $T_3 = \{0\} \times Z_{16} \times \{0\}\}$ be the subset semiring ideal of S.

V_4 = {Collection of all subsets from the subring $T_4 = (\{Z\} \times \{0\} \times Z_{15}) \subseteq \{Z \times Z_{16} \times Z_{15}\}\} \subseteq S$; be the subset semiring ideal of S.

S has subset idempotents for $A = \{(1, 1, 10)\} \in S$ is such that

$$\begin{aligned} A \times A &= \{(1, 1, 10)\} \times \{(1, 1, 10)\} \\ &= \{(1, 1, 10) \times (1, 1, 10)\} \\ &= \{(1, 1, 10)\} = A \text{ so A is a subset idempotent of S.} \end{aligned}$$

$B = \{(0, 1, 10)\} \in S$ is also a subset idempotent of S.

Example 2.48: Let S = {Collection of all subsets from the ring $P = R \times Z_{10} \times Z\}$ be the subset semiring. S has several subset subsemirings which are not subset semiring ideals of S.

M_1 = {Collection of all subsets from the subring $T_1 = \{3Z \times Z_{10} \times Z\} \subseteq P\}$ be the subset subsemiring of S. Clearly M_1 is not a subset semiring ideal.

Take M_2 = {Collection of all subsets from the subring $T_2 = \{5Z \times Z_{10} \times Z\} \subseteq P\}$ be the subset subsemiring of S.

Clearly M_2 is not a subset semiring ideal of S.

Let M_3 = {Collection of all subsets from the subring $T_3 = (16Z \times Z_{10} \times Z) \subseteq P\} \subseteq S$ is only a subset subsemiring of S which is not a subset semiring ideal of S.

We have infinite number of subset subsemirings in S which are not subset semiring ideals of S.

Infact S has also subset semiring ideals for take

W_1 = {Collection of all subsets from the subring $L_1 = R \times \{0\} \times \{0\} \subseteq P\} \subseteq S$ is the subset subsemiring which are also subset semiring ideals of S.

Let W_2 = {Collection of all subsets from the subring $L_2 = R \times Z_{16} \times \{0\} \subseteq P\} \subseteq S$ be the subset subsemiring which is also a subset semiring ideal of S.

Let W_3 = {Collection of all subsets from the subring $L_3 = \{R \times \{0\} \times 3Z\} \subseteq S\}$ be the subset subsemiring which is also a subset semiring ideal of S.

Infact S has infinite number of subset subsemirings which are subset semiring ideals of S.

Infact S has infinite number of subset zero divisors but only a finite number of subset idempotents.

Example 2.49: Let
S = {Collection of all subsets from the ring $R = Z_{11} \times Z_{19} \times Z_{23}$} be the subset semiring of finite order.

S has subset subsemirings which are subset ideals however S has no subset subsemiring which is not a subset semiring ideal.

However S has subset zero divisors and also subset idempotents.

Example 2.50: Let
S = {Collection of all subsets from the ring $Z \times Z_2 \times Z_7$} be the subset semiring. This has only the following subset zero divisors.

$A = \{(a, 0, 0)\}$ and $B = \{(0, a, b)\}$ in S are such that $A \times B = \{(0, 0, 0)\}$. However number of subset zero divisors is infinite in number.

The subset idempotents are $A_1 = \{(1, 0, 0)\}$, $A_2 = \{(0, 1, 0)\}$, $A_3 = \{(0, 0, 1)\}$, $A_4 = \{(1, 1, 0)\}$, $A_5 = \{(1, 0, 1)\}$ and $A_6 = \{(0, 1, 1)\}$ are the non trivial subset idempotents of S.

We see $\{(0, 0, 0)\}$ and $\{(1, 1, 1)\}$ are trivial subset idempotents of S.

Thus S has only finite number of subset idempotents. Also S has only finite number of subset units given by $B_1 = \{(1, 1, 2)\}$ and $B_2 = \{(1, 1, 4)\} \in S$ is such that $B_1 \times B_2 = \{(1, 1, 2)\} \times \{(1, 1, 4)\} = \{(1, 1, 1)\}$ is a subset unit of S.

$B_3 = \{(1, 1, 3)\}$ and $B_4 = \{(1, 1, 5)\}$ in S is such that $B_3 \times B_4 = \{(1, 1, 3)\} \times \{(1, 1, 5)\} = \{(1, 1, 1)\}$ is subset unit of S.

$B_5 = \{(1, 1, 6)\} \in S$ is such that $B_5 \times B_5 = \{(1, 1, 6)\} \times \{(1, 1, 6)\} = \{(1, 1, 1)\}$ is a subset unit of S. We have only 3 subset units in S though S is of infinite order.

Example 2.51: Let
S = {Collection of all subsets of the ring $R = Q \times Z_2 \times Z_7$} be the subset semiring. S has infinite number of subset units and finite number of subset idempotents and subset zero divisors.

However S has only finite number of subset semiring ideals but S has infinite number of subset subsemirings.

Example 2.52: Let
S = {Collection of all subsets from the ring $R = ZS_3 \times Z_2S_4 \times Z_7S_5$} be the subset semiring. S is non commutative has subset zero divisors and has infinite number of subset subsemiring which are not subset semiring ideals.

Now we give examples of infinite polynomial subset semirings of rings.

Example 2.53: Let
S = {Collection of all subsets from the polynomial ring Z[x]} be the subset semiring. S has no subset zero divisors and subset idempotents. However S has infinite number of subset subsemirings and subset semiring ideals.

Example 2.54: Let
S = {Collection of all subsets of the ring $Z_{12}[x]$} be the subset semiring. S has subset zero divisors, subset units and subset idempotents all of which are finite in number. S has also subset semiring ideals as well as subset subsemirings.

Example 2.55: Let
S = {Collection of all subsets from the polynomial ring $Z_{19}[x]$} be the subset semiring. S has no subset idempotents, no subset zero divisors. But S has infinite number of subset subsemiring which are not subset semiring ideals.

Example 2.56: Let
S = {Collection of all subset of the ring $(Z_2 \times Z_6 \times Z_{12})[x]$} be the subset semiring.

S has subset zero divisors. S has subset idempotents and subset units. S has infinite number of subset subsemiring and subset semiring ideals of finite number.

Now having seen examples of subsets of rings of all types we now proceed onto define subset interval semirings of type I and study them.

DEFINITION 2.2: *Let S = {Collection of all subsets from the interval ring; M = {[a, b] | a, b ∈ R; R a ring}}. S under the operations of R is a subset interval semiring defined as the subset semiring of type I.*

We will first illustrate this situation by some examples.

Example 2.57: Let S = {Collection of all subsets from the interval ring M = {[a, b] | a, b $\in$ Z_3}} be the subset interval semiring of type I. S is of finite order and is commutative.

Example 2.58: Let S = {Collection of all subsets from the interval ring M = {[a, b] | a, b $\in$ Z_{12}}} be the subset semiring. S is of finite order but S has subset zero divisors and subset idempotents.

Let A = {[6, 0]} $\in$ S. A^2 = {[6, 0]} × {[6, 0]} = {[0, 0]} so A is a subset zero divisor of S.

Let B = {[0, 4]} $\in$ S we see
B^2 = {[0, 4]} × {[0, 4]} = {[0, 4]} = B is a subset idempotent of S.

Take D = {[4, 0]} $\in$ S; we see
D × D = {[4, 0]} × {[4, 0]} = {[4, 0]} = D is a subset idempotent of S.

Take E = {[4, 4]} $\in$ S; we see E × E = E so E is a subset idempotent of S.

Also N_1 = {[0, 9]}, N_2 = {[9, 0]}, N_3 = {[9, 9]} in S are all subset idempotents of S. Further N_1 = {[4, 9]}, N_2 = {[9, 4]}, N_3 = {[0, 4], [4, 4], [4, 0], [0, 0]}, N_4 = {[0, 0], [9, 9], [9, 0], [0, 9]}, N_5 = {[0, 0], [4, 9], [9, 4], [0, 9], [0, 4], [4, 0], [9, 0], [9, 9], [4, 4]} $\in$ S are all subset idempotents of S.

Thus interval subset semiring has subset idempotents and subset zero divisors. Let V_1 = {Collection of all subsets from the interval ring

W_1 = {[a, b] | a, b $\in$ {0, 2, 4, 6, 8, 10} $\subseteq$ Z_{12}}} $\subseteq$ S be the interval subset semiring. V_1 is a subset interval subsemiring of S. V_1 also has subset interval subsemiring, subset interval idempotents and subset interval zero divisors. V_1 is also a subset interval semiring ideal of S.

Example 2.59: Let S = {Collection of all subsets from the interval ring M = {[a, b] | a, b $\in$ C(Z_{28})}} be the subset interval semiring.

S has subset subsemiring. T = {Collection of all subsets from the interval subring N = {[a, b] | a, b $\in$ {0, 2, 4, 6, …, 24, 26} $\subseteq$ C(Z_8)} $\subseteq$ M} $\subseteq$ S, is the subset interval subsemiring and T is also a subset semiring ideal of S.

Take W_1 = {Collection of all subsets from the interval subring L_1 = {[a, 0] | a $\in$ C(Z_{28})} $\subseteq$ M} $\subseteq$ S is again a subset interval subsemiring as well as subset semiring ideal of S.

W_2 = {Collection of all subsets from the interval subring L_2 = {[0, a] | a $\in$ C(Z_{28})} $\subseteq$ M} $\subseteq$ S be the subset interval subsemiring as well as subset interval semiring ideal of S.

We see W_1 and W_2 are isomorphic as subset interval subsemirings.

Example 2.60: Let S = {Collection of all subsets from the interval ring M = {[a, b] | a, b $\in$ Z(g_1, g_2) where $g_1^2 = 0$, $g_2^2 = g_2$, $g_1g_2 = g_2g_1 = 0$}} be the subset interval semiring of infinite order.

Clearly S has interval subset zero divisors, interval subset idempotents, interval subset subsemirings and interval subset semiring ideals.

Example 2.61: Let S = {collection of all subsets from the interval ring M = {[a, b] | a, b $\in$ $Z_8(g)$, where $g^2 = 0$}} be the subset interval semiring.

Take A = {[2, 4], [6, 0], [4, 4], [6, 2]} and
B = {[4, 2], [4, 0], [6, 2], [4, 4]} $\in$ S.

A × B = {[2, 4], [6, 0], [4, 4], [6, 2]} × {[4, 2], [4, 0], [6, 2], [4, 4]}

$= \{[2, 4] \times [4, 2], [2, 4] \times [4, 0], [2, 4] \times [6, 2], [2, 4] \times [4, 4], [6, 0] \times [4, 2], [6, 0] \times [4, 0], [6, 0] \times [6, 2], [6, 0] \times [4, 4], [4, 4] \times [4, 2], [4, 4] \times [4, 0], [4, 4] \times [6, 2], [4, 4] \times [4, 4], [6, 2] \times [4, 2], [6, 2] \times [4, 0], [6, 2] \times [6, 2], [6, 2] \times [4, 4]\}$

$= \{[0, 0], [4, 0], [0, 4], [4, 4]\} \in S.$

Now

$A + B = \{[2, 4], [6, 0], [4, 4], [6, 2]\} + \{[4, 2], [4, 0], [6, 2], [4, 4]\}$

$= \{[2, 4] + [4, 2], [2, 4] + [4, 0], [2, 4] + [6, 2], [2, 4] + [4, 4], [6, 0] + [4, 2], [6, 0] + [4, 0], [6, 0] + [6, 2], [6, 0] + [4, 4], [4, 4] + [4, 2], [4, 4] + [4, 0], [4, 4] + [6, 2], [4, 4] + [4, 4], [6, 2] + [4, 2], [6, 2] + [4, 0], [6, 2] + [6, 2], [6, 2] + [4, 4]\}$

$= \{[6, 6], [6, 4], [0, 6], [6, 0], [2, 2], [2, 0], [4, 2], [2, 4], [0, 4], [2, 6], [0, 0], [4, 4]\} \in S.$

This is the way operations on S are performed. However S is of finite order.

Example 2.62: Let S = {Collection of all subsets from the interval ring M = $\{[a, b] \mid a, b \in Q\}$} be the interval subset semiring of M. S is of infinite order. S has subset interval units but no subset interval idempotents.

S has trivial subset interval idempotents like $A_1 = \{[0,0]\}$, $A_2 = \{[1,1]\}$, $A_3 = \{[0,0], [1,1]\}$, $A_4 = \{[0,1]\}$, $A_5 = \{[1,0]\}$, $A_6 = \{[0,0], [1,0]\}$, $A_7 = \{[0,0], [0,1]\}$, $A_8 = \{[0,0], [1,0], [0,1]\}$ and $A_9 = \{[0,0], [1,1], [0,1], [1,0]\}$.

S has subset interval zero divisors. S has also subset interval subsemirings which are subset interval semiring ideals as well as subset interval subsemirings which are not subset interval semiring ideals.

Take P = {Collection of all subsets from the subset interval subring N = $\{[a, b] \mid a, b \in Z\} \subseteq M\} \subseteq S$ be the subset interval subsemiring of S. Clearly P is not a subset interval semiring ideal of S.

Further T = {Collection of all subsets from the interval subring L = $\{[a, 0] \mid a \in Q\} \subseteq M\} \subseteq S$ is again a subset interval subsemiring which is also a subset interval semiring ideal of S.

Take B = {[5, 0], [7/3, 0], [8, 0], [–7, 0]} and A = {[0, 5], [0, 11] [0, 17/5], [0, 5/3], [0, –10]} in S, we see A × B = {[0, 0]} infact S has infinite number of interval subset zero divisors. Likewise let A = {[7, 2]} and B = {[1/7, 1/2]} ∈ S, we see A × B = {[1, 1]} is the subset interval unit in S.

S has infinite number of subset unit intervals, however the cardinality of all the sets which contribute to subset interval units are only singleton sets.

A = {[3/2, 9/17]} and B = {[2/3, 17/9]} ∈ S is such that A × B = {[3/2, 9/17]} × {[2/3, 17/9]} = {[1, 1]}.

Example 2.63: Let S = {Collection of all subsets from the interval subring M = $\{[a, b] \mid a, b \in Z_5 \times Z_{12}\} = \{[(a_1, a_2), (b_1, b_2)] \mid a_1, b_1 \in Z_5 \text{ and } a_2, b_2 \in Z_{12}\}\}$ be the subset interval semiring of M. S has subset interval zero divisors and subset interval units. S is of finite order.

The operations of S are performed in this way.

If A = {[(3, 0), (2, 0)], [(2, 0), (4, 0)], [(0, 0), (1, 0)]} and B = {[(0, 7), (0, 2)], [(0, 9), (0, 0)], [(0, 6), (0, 5)]} ∈ S.

We find

$$\begin{aligned}A + B &= \{[(3, 0), (2, 0)], [(2, 0), (4, 0)], [(0, 0), (1, 0)]\} + \\ &\quad \{[(0, 7), (0, 2)], [(0, 9), (0, 0)], [(0, 6), (0, 5)]\} \\ &= \{[(3, 0), (2, 0)] + [(0, 7), (0, 2)], [(3, 0), (2, 0)] \\ &\quad + [(0, 9), (0, 0)], [(3, 0), (2, 0)] + [(0, 6), (0, 5)], \\ &\quad [(2, 0), (4, 0)] + [(0, 7), (0, 2)], [(2, 0), (4, 0)] + \\ &\quad [(0, 9), (0, 0)], [(2, 0), (4, 0)] + [(0, 6), (0, 5)], \\ &\quad [(0, 0), (1, 0)] + [(0, 7), (0, 2)], [(0, 0), (1, 0)] \\ &\quad + [(0, 9), (0, 0)], [(0, 0), (1, 0)] + [(0, 6),(0, 5)]\} \\ &= \{[(3, 7), (2, 2)], [(3, 9), (2, 0)], [(3, 6), (2, 5)], \\ &\quad [(2, 7), (4, 2)], [(2, 9), (4, 0)], [(2, 6), (4, 5)], \\ &\quad [(0, 6), (1, 5)], [(0, 9), (1, 0)], [(0, 7), (1, 3)]\} \\ &\quad \text{is in } S.\end{aligned}$$

This is the way operation + on S is performed.

In case of product in this case we see $A \times B = \{[(0, 0), (0, 0)]\}$ that is a subset interval zero divisor.

Let $A = \{[(3, 4), (2, 5)], [(3, 0), (3, 7)]\} \in S$.

$$\begin{aligned}A \times A &= \{[(3, 4), (2, 5)] \times [(3, 4), (2, 5)], [(3, 4), (2, 5)] \\ &\quad \times [(3, 0), (3, 7)], [(3, 0), (3, 7)] \times [(3, 0), (3, 7)], \\ &\quad [(3, 0), (3, 7)] \times [(3, 4), (2, 5)]\} \\ &= \{[(4, 4), (4, 1)], [(4, 0), (1, 11)], [(4, 0), (4, 1)]\} \\ &\qquad \in S.\end{aligned}$$

This is the way product is performed on S. S has subset interval subsemirings subset interval semiring ideals.

Example 2.64: Let S = {Collection of all subsets from the interval ring $M = \{[a, b] \mid a, b \in Z_{12} S_4\}$} be the subset interval semiring of M.

Let
$A = \{[5g_1 + 3g_2 + 1, 6g_3 + 8], [9g_{23} + 10, 11g_{20} + 6g_{12} + 1]\} \in S$ where $S_4 = \{e = 1, g_1, g_2, \ldots, g_{23}\}$. $A \times A \in S$.

$$\begin{aligned} A + A &= \{[5g_1 + 3g_2 + 1, 6g_3 + 8], [9g_{23} + 10, 11g_{20} + 6g_{12} + 1\} + \{[5g_1 + 3g_2 + 1, 6g_3 + 8], [9g_{23} + 10, 11g_{20} + 6g_{12} + 1]\} \\ &= \{[5g_1 + 3g_2 + 1, 6g_3 + 8] + [5g_1 + 3g_2 + 1, 6g_3 + 8], [5g_1 + 3g_2 + 1, 6g_3 + 8] + [9g_{23} + 10, 11g_{20} + 6g_{12} + 1], [9g_{23} + 10, 11g_{20} + 6g_{12} + 1] + [9g_{23} + 10, 11g_{20} + 6g_{12} + 1], [9g_{23} + 10, 11g_{20} + 6g_{12} + 1] + [5g_1 + 3g_2 + 1, 6g_3 + 8]\} \\ &= \{[10g_1 + 6g_2 + 2, 4], [9g_{23} + 5g_1 + 3g_2 + 5, 11g_{20} + 6g_3 + 6g_{12} + 9], [6g_{23} + 8, 10g_{20} + 2]\} \in S. \end{aligned}$$

This is the way operations are performed on S.

Example 2.65: Let S = {Collection of all subsets from the interval ring M = {[a, b] | a, b ∈ Z(S(3)), the semigroup ring of the symmetric group S(3) over the ring Z}} be the subset interval semiring.

We see for any

$$A = \left\{\left[3 + 5\begin{pmatrix}1 & 2 & 3\\ 2 & 1 & 3\end{pmatrix} + 10\begin{pmatrix}1 & 2 & 3\\ 3 & 2 & 1\end{pmatrix}, -4 + 6\begin{pmatrix}1 & 2 & 3\\ 2 & 3 & 1\end{pmatrix}\right]\right\} \text{ and}$$

$$B = \left\{\left[9-2\begin{pmatrix}1 & 2 & 3\\ 3 & 2 & 1\end{pmatrix}, 5 + 3\begin{pmatrix}1 & 2 & 3\\ 3 & 1 & 2\end{pmatrix}\right]\right\} \in S$$

Find

$$A + B = \{[3 + 5 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} + 10 \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}, -4 + 6 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}]\} + \{[9-2 \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}, 5 + 3 \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}]\}$$

$$= \{[12 + 5 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} + 8 \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}, 1 + 6 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} + 3 \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}]\} \text{ is in S.}$$

This is the way '+' operation is performed on S.

Now

$$A \times B = \{[3 + 5 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} + 10 \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}, -4 + 6 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}]\} \times \{[9-2 \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}, 5 + 3 \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}]\}$$

$$= \quad \{[3 + 5 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} + 10 \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}, -4 +$$

$$6 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}\Bigg] \times [9-2 \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}, 5 +$$

$$3 \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}\Bigg]\Bigg\}$$

$$= \quad \{[(3 + 5 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} + 10 \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}\Bigg) \times$$

$$([9-2 \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}\Bigg)\Bigg), \{-4 + 6 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}\Bigg) \times$$

$$(5 + 3 \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}\Bigg)\Bigg)\Bigg]\Bigg]\Bigg\}$$

$$= \{[27 + 45 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} + 90 \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} -$$

$$6 \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} - 10 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} - 20 \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix},$$

$$-20 + 30 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} - 12 \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} +$$

$$18 \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}\Bigg]\Bigg\}$$

$$= \quad \{[7 + 84 \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} - 10 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} +$$

$$45 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}, -2 + 30 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} -$$

$$12 \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}]\}.$$

This is the way the product operation × is performed on S.

S can have subset interval zero divisors, subset interval units, subset interval subsemirings and subset interval semiring ideals.

$$\text{Take } A = \left\{\left[\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}\right]\right\}$$

$$\text{and } B = \left\{\left[\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}\right]\right\} \in S.$$

Now

$$A \times B = \left\{\left[\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}\right]\right\} \times$$

$$\left\{\left[\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}\right]\right\}$$

$$= \left\{\left[\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}\right]\right\}$$

$$= \left\{\left[\begin{pmatrix}1 & 2 & 3\\1 & 2 & 1\end{pmatrix}, \begin{pmatrix}1 & 2 & 3\\1 & 2 & 3\end{pmatrix}\right]\right\}$$

which is the interval subset unit of S.

We have only finite number of interval subset units.

We call $\{[1, 1]\} = \left\{\left[\begin{pmatrix}1 & 2 & 3\\1 & 2 & 1\end{pmatrix}, \begin{pmatrix}1 & 2 & 3\\1 & 2 & 3\end{pmatrix}\right]\right\}$ as the interval subset identity in S with respect to product. Likewise $\{[0, 0]\} \in S$ is defined as the additive subset interval identity of S.

We see for every $A \in S$.

$$\{[0, 0]\} + A = A + \{[0, 0]\} = A \text{ and}$$
$$A \times \{[1, 1]\} = \{[1, 1]\} \times A = A \text{ for all } A \in S.$$

Now having seen examples of subset interval semirings of interval rings commutative or otherwise proceed on to describe more properties in case by using the basic interval ring as a non commutative ring.

Example 2.66: Let S = {Collection of all subsets from the interval group ring $M = \{[a, b] \mid a, b \in Z_{24} D_{2,9}$ where $D_{2,9} = \{a, b \mid a^2 = b^9 = 1, bab = a\}\}\}$ be the subset interval semiring of M. Clearly S is a non commutative subset interval semiring.

We can have subset interval right ideals in S which are not subset interval left ideals of S. Also S have subset interval semiring ideals.

If $A = \{[a, b]\}$ and $B = \{[b, a]\} \in S$.

$$\begin{aligned} A \times B &= \{[a, b]\} \times \{[b, a]\} \\ &= \{\{[a, b] \times [b, a]\} \\ &= \{[ab, ba]\} \qquad \ldots \qquad \text{I} \end{aligned}$$

Consider

$$\begin{aligned} B \times A &= \{[b, a]\} \times \{[a, b]\} \\ &= \{[b, a] \times [a, b]\} \\ &= \{[ba, ab]\} \qquad \ldots \qquad \text{II} \end{aligned}$$

Clearly I and II are distinct as $A \times B \neq B \times A$.

Thus S is non commutative subset interval semiring.

If $A = \{[a, b]\}$ and $B = \{[a, b^8]\} \in S$ we have

$$\begin{aligned} A \times B &= \{[a, b]\} \times \{[a, b^8]\} \\ &= \{[a, b] \times [a, b^8]\} \\ &= \{[a^2, b^9]\} = \{[1, 1]\} \text{ as } a^2 = 1 \text{ and } b^9 = 1. \end{aligned}$$

Let $A = \{[1, b^5]\}$ and $B = \{[1, b^4]\} \in S$; now
$A \times B = \{[1, b^5]\} \times \{[1, b^4]\} = \{[1, b^9]\} = \{[1, 1]\}$
(as $b^9 = 1$)

Thus we have subset interval units in S.

Further if $A = \{[0, 1 + b^2 + ab]\}$ and
$B = \{[4 + ab + ab^2, 0]\} \in S$.

$$\begin{aligned} \text{We get } A \times B &= \{[0, 1 + b^2 + ab]\} \times \{[4 + ab + ab^2 , 0]\} \\ &= \{[0 \times 4 + ab + ab^2,\ 1 + b^2 + ab \times 0]\} \\ &= \{[0, 0]\} \end{aligned}$$

is a subset interval zero divisor of S.

Example 2.67: Let S = {Collection of all subsets from the interval ring M = {[a, b] | a, b ∈ $Z_2 S_3$}} be the subset interval semiring. S has subset interval units, subset interval idempotents and subset interval zero divisors. Further S has subset interval subsemirings and subset interval semiring ideals.

Now having seen examples of all these we now proceed on to describe interval matrix rings by some examples.

Example 2.68: Let S = {Collection of all subsets from the interval ring M = {[a, b] | a, b ∈ $\langle Z_5 \cup I \rangle$}} be the subset interval semiring. S has subset interval zero divisors.

S has subset interval units and S has subset interval subsemirings and S has subset interval idempotents. S has subset interval subsemirings that are subset interval semiring ideals.

Now having seen examples of interval subset semirings of interval rings; we now proceed onto describe subset interval semirings of interval matrix rings.

Example 2.69: Let S = {Collection of all subsets from the interval matrix ring M = {([a_1, b_1], [a_2, b_2], [a_3, b_3], [a_4, b_4]) | a_i, b_i ∈ Z_{12}; $1 \leq i \leq 4$}} be the subset interval matrix semiring of the interval matrix ring M.

S has subset interval zero divisors, subset interval idempotents, subset interval units, subset interval subsemirings and subset interval semiring ideals.

Example 2.70: Let S = {Collection of all subsets from the interval matrix ring M = {([a_1, b_1], [a_2, b_2]) | a_i, b_i ∈ Z_6; $1 \leq i \leq 2$}} be the subset interval matrix semiring.

Let A = {([2, 0], [0, 4]), ([4, 0], [1, 2])} and
B = {([4, 3], [1, 5])} ∈ S.

Now

$$\begin{aligned} A + B &= \{([2, 0], [0, 4]), ([4, 0], [1, 2])\} + \{([4, 3], [1, 5])\} \\ &= \{([2, 0], [0, 4]) + ([4, 3], [1, 5]), ([4, 0], [1, 2]) + ([4, 3], [1, 5])\} \\ &= \{([0, 3], [1, 3]), ([2, 3], [2, 1])\} \in S. \end{aligned}$$

We find

$$\begin{aligned} A \times B &= \{([2, 0], [0, 4]), ([4, 0], [1, 2])\} \times \{([4, 3], [1, 5])\} \\ &= \{([2, 0], [0, 4]) \times ([4, 3], [1, 5]), ([4, 0], [1, 2]) \times ([4, 3], [1, 5])\} \\ &= \{([2, 0] \times [4, 3], [0, 4] \times [1, 5]), ([4, 0] \times [4, 3], [1, 2] \times [1, 5])\} \\ &= \{([2, 0], [0, 2]), ([4, 0], [1, 4])\} \text{ is in } S. \end{aligned}$$

This is the way operations on S is performed.

We see S has subset interval zero divisors.

Take $A = \{([3, 0], [0, 4])\}$ and $B = \{[0, 5], [2, 0])\} \in S$.

We see $A \times B = \{([0, 0], [0, 0])\}$ so A and B are subset interval zero divisors. ([0, 0], [0, 0]) is defined as the subset interval zero or subset interval additive identity.

Similarly ([1, 1], [1, 1]) in S is the multiplicative subset interval identity of S.

For take $A = \{(([5, 1], [1, 5])\} \in S$.

Further $A \times A = \{([1, 1], [1, 1])\}$; thus A is a subset interval unit of S.

B = {([5, 5], [1, 1])} ∈ S, we find B × B = {([1, 1], [1, 1])} is the subset interval unit of S.

Example 2.71: Let S = {Collection of all subsets from the interval matrix ring

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ [a_3, b_3] \end{bmatrix} \right| a_i, b_i \in Z_7, 1 \leq i \leq 3 \right\}\}$$

be the subset interval semiring of the interval matrix ring M.

We see S has subset interval zero divisors and subset interval idempotents and so on.

Take $X = \left\{ \begin{bmatrix} [3,1] \\ [1,2] \\ [6,1] \end{bmatrix} \right\}$ and $Y = \left\{ \begin{bmatrix} [5,1] \\ [1,4] \\ [6,1] \end{bmatrix} \right\} \in S$.

$$\text{Now } X \times Y = \left\{ \begin{bmatrix} [3,1] \\ [1,2] \\ [6,1] \end{bmatrix} \right\} \times \left\{ \begin{bmatrix} [5,1] \\ [1,4] \\ [6,1] \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} [3,1] \\ [1,2] \\ [6,1] \end{bmatrix} \times \begin{bmatrix} [5,1] \\ [1,4] \\ [6,1] \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} [1,1] \\ [1,1] \\ [1,1] \end{bmatrix} \right\} \in S.$$

Further X and Y are inverses of each other that is they are subset interval units of S.

$\left\{\begin{bmatrix}[1,1]\\[1,1]\\[1,1]\end{bmatrix}\right\}$ is the subset interval unit of S.

We now give some subset interval zero divisors of S.

Let A = $\left\{\begin{bmatrix}[3,0]\\[6,0]\\[0,2]\end{bmatrix}\right\}$ and B = $\left\{\begin{bmatrix}[0,5]\\[0,1]\\[6,0]\end{bmatrix}\right\} \in$ S.

$$A \times B = \left\{\begin{bmatrix}[3,0]\\[6,0]\\[0,2]\end{bmatrix}\right\} \times \left\{\begin{bmatrix}[0,5]\\[0,1]\\[6,0]\end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix}[3,0]\times[0,5]\\[6,0]\times[0,1]\\[0,2]\times[6,0]\end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix}[0,0]\\[0,0]\\[0,0]\end{bmatrix}\right\}.$$

Thus S has subset interval zero divisors and $\left\{\begin{bmatrix}[0,0]\\[0,0]\\[0,0]\end{bmatrix}\right\}$ is the subset interval zero of S.

Consider $A = \left\{ \begin{bmatrix} [3,4] \\ [0,5] \\ [6,1] \end{bmatrix} \right\}$ and $B = \left\{ \begin{bmatrix} [4,3] \\ [0,2] \\ [1,6] \end{bmatrix} \right\} \in S$.

$$A + B = \left\{ \begin{bmatrix} [3,4] \\ [0,5] \\ [6,1] \end{bmatrix} \right\} + \left\{ \begin{bmatrix} [4,3] \\ [0,2] \\ [1,6] \end{bmatrix} \right\} = \left\{ \begin{bmatrix} [0,0] \\ [0,0] \\ [0,0] \end{bmatrix} \right\}.$$

Example 2.72: Let S = {Collection of all subsets from the interval matrix ring

$$M = \left\{ \begin{bmatrix} [a_1,b_1] & [a_2,b_2] \\ [a_3,b_3] & [a_4,b_4] \\ [a_5,b_5] & [a_6,b_6] \\ [a_7,b_7] & [a_8,b_8] \end{bmatrix} \middle| \, a_i, b_i \in Z, 1 \leq i \leq 8 \right\}\}$$

be the subset interval matrix semiring.

Clearly S is a subset interval matrix zero divisors but S has no non trivial subset interval matrix units in S.

We give some subset interval zero divisors of S.

Take $A = \left\{ \begin{bmatrix} [0,1] & [6,0] \\ [0,0] & [7,0] \\ [-1,1] & [-1,0] \\ [8,0] & [-7,0] \end{bmatrix} \right\}$ and

$$B = \left\{\begin{bmatrix} [1,0] & [0,8] \\ [8,-19] & [0,25] \\ [0,0] & [0,-28] \\ [0,-15] & [0,25] \end{bmatrix}\right\}$$

$$A \times B \;= \left\{\begin{bmatrix} [0,0] & [0,0] \\ [0,0] & [0,0] \\ [0,0] & [0,0] \\ [0,0] & [0,0] \end{bmatrix}\right\} \in S \text{ is such that}$$

$A \times B$ is a subset interval zero divisor of S.

Example 2.73: Let S = {Collection of all subsets from the interval matrix ring

$$M = \left\{\begin{bmatrix} [a_1,b_1] & [a_2,b_2] \\ [a_3,b_3] & [a_4,b_4] \end{bmatrix} \middle| \; a_i, b_i \in Z_{36}, 1 \leq i \leq 4\right\}\}$$

be the subset interval matrix semiring of M.

S has subset interval zero divisors, S has subset interval idempotents, S has subset interval units, S has subset interval subsemiring and subset interval semiring ideals.

Take

$$P = \left\{\begin{bmatrix} [a_1,b_1] & [a_2,b_2] \\ [a_3,b_3] & [a_4,b_4] \end{bmatrix} \middle| \; a_i, b_i \in 2Z_{36} = \{0, 2, 4, 6, 8, \ldots, 34\}, 1 \leq i \leq 4\right\} \subseteq M.$$

T = {(Collection of all subsets from the interval matrix subring P} ⊆ S is a subset interval subsemiring and T is also a subset interval semiring ideal of S.

Let A = $\left\{\begin{bmatrix} [0,6] & [7,2] \\ [4,5] & [3,0] \end{bmatrix}\right\}$ and B = $\left\{\begin{bmatrix} [2,4] & [9,2] \\ [1,2] & [8,6] \end{bmatrix}\right\}$ ∈ S.

We find both A + B and A × B.

Consider

$$A + B = \left\{\begin{bmatrix} [0,6] & [7,2] \\ [4,5] & [3,0] \end{bmatrix}\right\} + \left\{\begin{bmatrix} [2,4] & [9,2] \\ [1,2] & [8,6] \end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix} [0,6]+[2,4] & [7,2]+[9,2] \\ [4,5]+[1,2] & [3,0]+[8,6] \end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix} [2,10] & [16,4] \\ [5,7] & [11,6] \end{bmatrix}\right\} \in S.$$

$$A \times B = \left\{\begin{bmatrix} [0,6] & [7,2] \\ [4,5] & [3,0] \end{bmatrix}\right\} \times \left\{\begin{bmatrix} [2,4] & [9,2] \\ [1,2] & [8,6] \end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix} [0,6]\times[2,4] & [7,2]\times[9,2] \\ [4,5]\times[1,2] & [3,0]\times[8,6] \end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix} [0,24] & [27,4] \\ [4,10] & [24,0] \end{bmatrix}\right\} \in S.$$

This is the way operations are performed on S.

Clearly S is a commutative subset interval semiring of finite order.

Take A = $\left\{\begin{bmatrix} [6,12] & [3,0] \\ [4,9] & [12,3] \end{bmatrix}\right\}$

$$\text{and } B = \left\{\begin{bmatrix} [6,3] & [12,0] \\ [9,4] & [3,12] \end{bmatrix}\right\} \in S.$$

$$A \times B = \left\{\begin{bmatrix} [0,0] & [0,0] \\ [0,0] & [0,0] \end{bmatrix}\right\};$$

thus A is a subset interval zero divisor of S.

Now consider $A = \left\{\begin{bmatrix} [35,1] & [35,35] \\ [1,1] & [1,35] \end{bmatrix}\right\} \in S$

$A \times A = \left\{\begin{bmatrix} [1,1] & [1,1] \\ [1,1] & [1,1] \end{bmatrix}\right\}$; thus A is the subset interval unit of S.

$A = \left\{\begin{bmatrix} [0,1] & [1,1] \\ [0,1] & [1,0] \end{bmatrix}\right\} \in S$ is such that $A \times A = A$ is a subset interval idempotent of S.

Thus S has subset interval idempotents.

Example 2.74: Let S = {Collection of all subsets from the interval matrix ring

$$M = \left\{\begin{bmatrix} [a_1,b_1] & [a_2,b_2] & [a_3,b_3] & [a_4,b_4] \\ [a_5,b_5] & [a_6,b_6] & [a_7,b_7] & [a_8,b_8] \end{bmatrix} \middle| \; a_i, b_i \in Z_{15}, \right.$$

$$1 \leq i \leq 8\}\}$$

be the subset interval matrix semiring.

It is easily verified S has subset interval units, subset interval idempotents, subset interval zero divisors, subset interval subsemirings and subset interval semiring ideals.

We can have subset interval matrix semirings both of finite and infinite order.

We can have subset interval matrix semirings both commutative and non commutative. We will provide a few examples of non commutative subset matrix interval semirings.

Example 2.75: Let S = {Collection of all subsets from the interval matrix ring

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ [a_3, b_3] \\ [a_4, b_4] \\ [a_5, b_5] \end{bmatrix} \right| a_i, b_i \in Z_4S_3; 1 \leq i \leq 5 \right\} \}$$

be the subset interval matrix semiring.

Clearly S is non commutative subset interval matrix semiring of finite order.

S has subset interval zero divisors.

S in general is non commutative.

$$\text{Take } A = \left\{ \begin{bmatrix} [0, g_1] \\ [g, g_2] \\ [g_2, 0] \\ [g_3, 1] \\ [g_1, g_3] \end{bmatrix} \right\} \text{ and } B = \left\{ \begin{bmatrix} [0, g_2] \\ [g_2, g_3] \\ [g_3, 0] \\ [0, g_3] \\ [g_2, g_4] \end{bmatrix} \right\} \in S.$$

We find A × B

$$= \left\{\begin{bmatrix} [0, g_1] \\ [g, g_2] \\ [g_2, 0] \\ [g_3, 1] \\ [g_1, g_3] \end{bmatrix}\right\} \times \left\{\begin{bmatrix} [0, g_2] \\ [g_2, g_3] \\ [g_3, 0] \\ [0, g_3] \\ [g_2, g_4] \end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix} [0, g_1] \times [0, g_2] \\ [g, g_2] \times [g_2, g_3] \\ [g_2, 0] \times [g_3, 0] \\ [g_3, 1] \times [0, g_3] \\ [g_1, g_3] \times [g_2, g_4] \end{bmatrix}\right\}$$

(Here $g_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$, $g_2 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$, $g_3 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$,

$g_4 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$, $g_5 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}\Big\}$

$$= \left\{\begin{bmatrix} [0, g_5] \\ [g_5, g_5] \\ [g_5, 0] \\ [0, g_3] \\ [g_5, g_2] \end{bmatrix}\right\} \qquad \ldots \quad \text{I}$$

Now we find B × A

$$= \left\{ \begin{bmatrix} [0, g_2] \\ [g_2, g_3] \\ [g_3, 0] \\ [0, g_3] \\ [g_2, g_4] \end{bmatrix} \right\} \times \left\{ \begin{bmatrix} [0, g_1] \\ [g, g_2] \\ [g_2, 0] \\ [g_3, 1] \\ [g_1, g_3] \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} [0, g_2] \times [0, g_1] \\ [g_2, g_3] \times [g, g_2] \\ [g_3, 0] \times [g_2, 0] \\ [0, g_3] \times [g_3, 1] \\ [g_2, g_4] \times [g_1, g_3] \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} [0, g_4] \\ [g_4, g_4] \\ [g_4, 0] \\ [0, g_3] \\ [g_4, g_1] \end{bmatrix} \right\} \qquad \ldots \text{ II}$$

I and II are distinct.

Thus A × B $\neq$ B × A.

So S is not a commutative subset matrix semiring.
Now we find

A + B

$$= \left\{ \begin{bmatrix} [0, g_1] \\ [g, g_2] \\ [g_2, 0] \\ [g_3, 1] \\ [g_1, g_3] \end{bmatrix} \right\} + \left\{ \begin{bmatrix} [0, g_2] \\ [g_2, g_3] \\ [g_3, 0] \\ [0, g_3] \\ [g_2, g_4] \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} [0, g_1 + g_2] \\ [g_1 + g_2, g_2 + g_3] \\ [g_2 + g_3, 0] \\ [g_3, 1 + g_3] \\ [g_1 + g_2, g_3 + g_4] \end{bmatrix} \right\}.$$

This is the way '+' operation is performed on S.

Example 2.76: Let S = {Collection of all subsets from the matrix interval ring

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \\ \vdots & \vdots \\ [a_{23}, b_{23}] & [a_{24}, b_{24}] \end{bmatrix} \middle| \; a_i, b_i \in QS_7 \; ; \; 1 \leq i \leq 24 \right\}\}$$

be the subset interval matrix semiring of infinite order which is non commutative.

This has interval subset units, interval subset zero divisors, interval subset idempotents, interval subset subsemirings and interval subset semiring ideals.

Example 2.77: Let S = {Collection of all subsets from the interval matrix ring

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] & \ldots & [a_6, b_6] \\ [a_7, b_7] & [a_8, b_8] & \ldots & [a_{12}, b_{12}] \end{bmatrix} \middle| \; a_i, b_i \in (\langle Q \cup I \rangle D_{2,13}, \; 1 \leq i \leq 12 \right\}\}$$

be the subset interval matrix semiring of M.

S has interval subset units, interval subset idempotents, interval subset zero divisors, interval subset subsemirings and interval subset semiring ideals.

Clearly S is non commutative of infinite order.

Example 2.78: Let S = {Collection of all subsets from the interval matrix ring

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] & [a_3, b_3] \\ [a_4, b_4] & [a_5, b_5] & [a_6, b_6] \\ [a_7, b_7] & [a_8, b_8] & [a_9, b_9] \end{bmatrix} \middle| \; a_i, b_i \in (Z_5 \times Z_7 \times Z_{11}) \right.$$

$$(S(5)); 1 \leq i \leq 9\}\}$$

be the subset interval semiring.

S has subset interval matrix subsemiring, subset interval zero divisors, subset interval units and subset interval idempotents.

Example 2.79: Let S = {Collection of all subsets from the interval matrix ring

$$M = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{10}, b_{10}] \end{bmatrix} \middle| \; a_i, b_i \in Z(S_3 \times A_4); 1 \leq i \leq 10\}\} \right.$$

be the subset interval semiring which is of infinite order but non commutative has infinite number of subset interval zero divisors.

Now having seen examples of finite and infinite commutative and non commutative subset interval matrix semirings we now proceed onto study subset interval polynomial semirings built over interval polynomial rings.

Example 2.80: Let S = {Collection of all subsets from the interval polynomial ring

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in Z_5[x] \right\}\}$$

be the subset interval polynomial semiring.

Clearly S is commutative and is of infinite order.

Let $A = \{[3, 0]x^3 + [2, 1]x^2 + [3, 0]x\}$ and
$B = \{[2, 4]x^2 + [1, 0]\} \in S$.

We find

A + B

$= \{[3, 0]x^3 + [2, 1]x^2 + [3, 0]x\} + \{[2, 4]x^2 + [1, 0]\}$

$= \{[3, 0]x^3 + [2, 1] + [2, 4]x^2 + [1, 0], [4, 1]x^2 + [3, 0]x + [2, 4]\, x^2 + [1, 0]\}$

$= \{[3, 0]x^3 + [2, 4]x^2 + [3, 1], [1, 0]x^2 + [3, 0]x + [1, 0]\}$ is in S.

This is the way the operation of "+' is performed on S.

We find

A × B

$= \{[3, 0]x^3 + [2, 1]x^2 + [3, 0]x\} \times \{[2, 4]x^2 + [1, 0]\}$

$= \{([3, 0]x^3 + [2, 1]) \times ([2, 4]x^2 + [1, 0])\, ([4,1]x^2 + [3, 0]x)\, ([2, 4]x^2 + [1, 0])\}$

$= \{([3, 0] \times [2, 4]\, x^5 + [2, 1] \times [2, 4]x^2 + [3, 0] \times [1, 0]x^3 + [2, 1] \times [1, 0]), [4, 1] \times [2, 4]\, x^4 + [3, 0] \times [2, 4]x^3 + [4, 1] \times [1, 0]\, x^2 + [3, 0]\, [1, 0]x\}$

$= \{[1, 0]\, x^5 + [4, 4]x^2 + [3, 0]x^3 + [2, 0], [3, 4]x^4 + [1, 0]x^3 + [4, 0]x^2 + [3, 0]x\} \in S.$

This is the way operation of product is performed on S.

We find subset interval polynomial subsemirings of S.

Take P_1 = {Collection of all subsets from the interval polynomial subring

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, 0]x^i \,\middle|\, a_i \in Z_5[x]\right\}\} \subseteq S$$

be the subset interval polynomial subsemiring of S which is also a subset interval polynomial semiring ideal of S.

Now P_2 = {Collection of all subsets from the interval polynomial subring

$$M_2 = \left\{ \sum_{i=0}^{\infty} [0, a_i]x^i \,\middle|\, a_i \in Z_5[x]\right\} \subseteq M\} \subseteq S$$

be the subset interval polynomial subsemiring of S. P_2 is a subset interval polynomial semiring ideal of S.

We see $P_1 \times P_2 = \{[0, 0]\}$. Thus they annihilate each other.

In view of this we see we have an infinite collection of subset interval polynomial zero divisors.

For every $A \in P_1$, and $B \in P_2$ we have $A \times B = \{[0, 0]\}$.

Example 2.81: Let S = {Collection of all subsets from the interval polynomial ring

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in Z_{12}[x] \right\}\}$$

be the subset interval polynomial semiring. S has infinite number of subset interval polynomial zero divisors.

Infact S has two interval subset polynomial semiring ideals P_1 and P_2 with $P_1 \times P_2 = \{[0, 0]\}$ both of them are of infinite order.

Example 2.82: Let S = {Collection of all subsets from the interval polynomial ring

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in Z[x] \right\}\}$$

be the subset interval polynomial semiring of infinite order. This S also has infinite number of subset interval polynomial zero divisors.

This S also has two subset interval polynomial semiring ideals P_1 and P_2 with $P_1 \times P_2 = \{[0, 0]\}$ and $P_1 \cap P_2 = \{[0, 0]\}$. This sort of ideals are also prevalent in subset interval matrix semirings.

Example 2.83: Let S = {Collection of all subsets from the matrix ring

$$M = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_9, b_9] \end{bmatrix} \,\middle|\, a_i, b_i \in Z_{16}; 1 \leq i \leq 9 \right\}\}$$

be the subset interval matrix semiring.

Take P_1 = {Collection of all subsets from the interval matrix subring

$$M_1 = \left\{ \left. \begin{bmatrix} [a_1, 0] \\ [a_2, 0] \\ \vdots \\ [a_9, 0] \end{bmatrix} \right| a_i \in Z_{16};\ 1 \leq i \leq 6\} \subseteq M\} \subseteq S.$$

Clearly P_1 is a subset interval matrix subsemiring which is also a subset interval matrix semiring ideal of S.

Consider P_2 = {Collection of all subsets from the interval matrix subring

$$M_2 = \left\{ \left. \begin{bmatrix} [0, b_1] \\ [0, b_2] \\ \vdots \\ [0, b_9] \end{bmatrix} \right| a_i, b_i \in Z_{16};\ 1 \leq i \leq 6\} \subseteq M\} \subseteq S$$

be the subset interval matrix subsemiring.

P_2 is also a subset interval matrix semiring ideal of S. Thus $P_1 \times P_2 = \{[0, 0]\}$ and $P_1 \cap P_2 = \{[0, 0]\}$.

Example 2.84: Let S = {Collection of all subsets from the interval matrix ring $M = \{([a_1, b_1], \ldots, [a_{18}, b_{18}]) \mid a_i, b_i \in Z_{43}, 1 \leq i \leq 18\}$ be the subset interval matrix semiring of the interval matrix ring M.

P_1 = {Collection of all subsets from the interval matrix subring $M_1 = \{([a_1, 0], \ldots, [a_{18}, 0]) \mid a_i \in Z_{43}, 1 \leq i \leq 18\} \subseteq M\} \subseteq$ S be the subset interval matrix subsemiring which is also a subset interval matrix semiring ideal of S.

$P_1 \times P_2 = \{[0, 0]\}$ and $P_1 \cap P_2 = \{[0, 0]\}$.

Example 2.85: Let S = {Collection of all susbets from the interval matrix ring

$$M = \left\{\left.\begin{bmatrix} [a_1, b_1] & \ldots & [a_{10}, b_{10}] \\ [a_{11}, b_{11}] & \ldots & [a_{20}, b_{20}] \\ [a_{21}, b_{21}] & \ldots & [a_{30}, b_{30}] \end{bmatrix}\right| a_i, b_i \in Z_{144}; 1 \leq i \leq 30\}\}$$

be the subset interval matrix semiring.

Let P_1 = {Collection of all subsets from the interval matrix subring

$$M_1 = \left\{\left.\begin{bmatrix} [a_1, 0] & \ldots & [a_{10}, 0] \\ [a_{11}, 0] & \ldots & [a_{20}, 0] \\ [a_{21}, 0] & \ldots & [a_{30}, 0] \end{bmatrix}\right| a_i \in Z_{144}; 1 \leq i \leq 30\} \subseteq M\} \subseteq S$$

be the subset interval matrix subsemiring of S which is also a subset interval matrix semiring ideal of S.

Consider P_2 = {Collection of all subsets from the interval matrix subring

$$M_2 = \left\{\left.\begin{bmatrix} [0, a_1] & \ldots & [0, a_{10}] \\ [0, a_{11}] & \ldots & [0, a_{20}] \\ [0, a_{21}] & \ldots & [0, a_{30}] \end{bmatrix}\right| a_i \in Z_{144}; 1 \leq i \leq 30\} \subseteq M\} \subseteq S$$

be the subset interval matrix subsemiring of S.

P_2 is also a subset interval matrix semiring ideal of S.

$P_1 \times P_2 = \{[0,0]\}$ leading to infinite number of subset interval matrix zero divisors. Further $P_1 \cap P_2 = \{[0,0]\}$.

In view of all these we have the following interesting results.

THEOREM 2.5: *Let S = {Collection of all susbets of the interval matrix ring M = {A = ($[a_{ij}, b_{ij}]$) | $a_{ij}, b_{ij} \in R$; R a ring or a field and A is a $n \times m$ interval matrix $1 \leq i \leq n$ and $1 \leq j \leq m$}} be the subset interval matrix semiring of M.*

S has atleast two subset interval matrix semiring ideals say P_1 and P_2 such that $P_1 \times P_2$ = {$([0, 0])_{n\times n}$; zero matrix} = $P_1 \cap P_2$.

Proof is direct and hence left as an exercise to the reader.

THEOREM 2.6: *Let S = {Collection of all subsets from the interval polynomial ring*

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \right| a_i, b_i \in R \text{ a ring}\}\}$$

be the subset interval polynomial semiring. S has atleast two subset interval polynomial semiring ideals. P_1 and P_2 such that $P_1 \times P_2$ = {[0,0]} and $P_1 \cap P_2$ = {[0,0]}.

The proof is direct and hence left as an exercise to the reader.

We have given examples to this effect.

We now give examples of subset polynomial interval semiring of finite order.

Let us define in a polynomial ring in the variable x in which we take $x^n = 1$; n a finite an integer.

Example 2.86: Let S = {Collection of all subsets from the interval polynomial ring

$$M = \left\{ \sum_{i=0}^{n} [a_i, b_i]x^i \;\middle|\; a_i, b_i \in Z_{24}; 0 \leq i \leq n \text{ and } x^{n+1} = 1; n < \infty \right\}\}$$

be the subset interval polynomial semiring of M. Clearly S is of finite order.

Even in this S we have two distinct subset interval polynomial semiring ideals using

$$M_1 = \left\{ \sum_{i=0}^{n} [a_i, 0]x^i \;\middle|\; a_i, 0 \in Z_{24} \right\} \subseteq M$$

where P_1 = {Collection of all subsets from the interval polynomial subsemiring of M_1} ⊆ S and P_2 = {Collection of all subsets from the interval polynomial subsemiring

$$M_2 = \left\{ \sum_{i=0}^{n} [0, b_i]x^i \;\middle|\; b_i \in Z_{24} \right\} \subseteq M\} \subseteq S;$$

both P_1 and P_2 are subset interval polynomial semiring ideals of S.

$o(P_1) = o\,(P_2) < \infty$ with $P_1 \times P_2 = \{[0, 0]\}$ and $P_1 \cap P_2 = \{[0, 0]\}$.

This is the way we can construct finite subset interval polynomial semirings.

Example 2.87: Let S = {Collection of all subsets from the interval polynomial ring

$$M = \left\{ \sum_{i=0}^{5} [a_i, b_i]x^i \;\middle|\; a_i, b_i \in Z_{12}, 0 \leq i \leq 5, x^6 = 1 \right\}\}$$

be the subset interval polynomial semiring of finite order. S has subset interval zero divisors, subset interval idempotents and subset interval units.

S has subset interval polynomial subsemirings as well as subset interval polynomial semiring ideals.

Example 2.88: Let S = {Collection of all subsets from the interval polynomial

$$M = \left\{ \sum_{i=0}^{7} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in Z_6S_3, 0 \leq i \leq 7, x^8 = 1 \right\}\}$$

be the subset interval polynomial semiring of finite order. S is a non commutative interval polynomial semiring.

This S also has subset interval units, subset interval idempotents, subset interval zero divisors, subset interval polynomial subsemiring and subset interval polynomial semiring ideals.

$$\text{Let } A = \{[2\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}, 3\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix} + \begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}]x +$$

$$[0, 4\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}], [4, 3\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix} + 4 +$$

$$2\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}]x + [5, 0]\}$$

and

$$B = \{[3\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix} + \begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}, 4 + 2\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}] x^2$$

$$+ \left[\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}, \begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}\right]\} \in S.$$

First we find

$$A + B \;=\; \{[2\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}, 3\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} +$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}]x + [0, 4\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}],$$

$$[4, 3\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} + 4 + 2\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}]x +$$

$$[5, 0]\} + \{[3\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} + 5\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix},$$

$$4 + 2\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}]x^2 + [\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix},$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}]\}$$

$$= \{[2\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}, 3\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} +$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}]x + [3\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}] +$$

$$5\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}, 4 + 2\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}]x^2$$

$$+ [\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}, 4\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}]$$

$$= [3\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix} + 5\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}, 4 +$$

$$2\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}]x^2 + [4, 3\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix} + 4 +$$

$$2\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}]x + [5 + \begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}, \begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}]\}$$

$$\in S.$$

This is the way addition is performed in S.

We find

$$A \times B \quad = \quad \{[2\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}, 3\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix} + \begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}]x +$$

$$[0, 4\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}], [4, 3\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix} + 4 +$$

$$2\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}]x + [5, 0]\} \times \{[3\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix} +$$

$$5\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}, 4 + 2\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}]x^2 +$$

$$[\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}, \begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}]\}$$

$$= \quad \{[2\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}, 3\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix} + \begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}]x \times$$

$$[3 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} + 5 \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}, 4 +$$

$$2 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}] x^2 + [0, 4 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}] \times$$

$$[3 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} + 5 \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}, 4 +$$

$$2 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}] x^2 + \{[2 \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}, 3 \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$$

$$+ \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}] x \times [\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}] +$$

$$[0, 4 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}] \times [\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}],$$

$$[4, 3 \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} + 4 + 2 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}] x \times$$

$$[3 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} + 5 \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}, 4 +$$

$$2 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}] x^2 + [5, 0] [\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix},$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}] + [5, 0] \times [3 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} +$$

$$5 \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}, 4 + 2 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \Bigg] x^2$$

$$+ [4, 3 \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} + 4 + 2 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \Bigg] \times$$

$$\Bigg[\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \Bigg] x \}$$

$$= \quad \{[4 \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}, 4 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} + 2 \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \Bigg] x^3 +$$

$$[0, 4 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} + 2 \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \Bigg] x^2 + [2 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix},$$

$$3 \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \Bigg] x + [0, 4 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \Bigg],$$

$$[2 \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}, 4 + 2 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} + 2 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} +$$

$$4 \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \Bigg] x^3 + [5 \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}, 0] + [3 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$$

$$+ \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}, 0] x^2 + [4 \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}, 3 \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} +$$

$$4 \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} + 2 \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \Bigg] x\} \in S.$$

This is the way product is performed on S.

It is left as an exercise for the reader to prove that in S. $A \times B \neq B \times A$, in general for $A, B \in S$.

Example 2.89: Let S = {Collection of all subsets from the interval polynomial ring

$$M = \left\{ \sum_{i=0}^{9} [a_i, b_i] x^i \;\middle|\; a_i, b_i \in Z_{12} D_{2,9}, x^{10} = 1, 0 \leq i \leq 9 \right\}\}$$

be the subset interval polynomial semiring of finite order, S is clearly non commutative.

Example 2.90: Let S = {Collection of all subsets from the interval polynomial ring

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \;\middle|\; a_i, b_i \in C(\langle Z_{10} \cup I \rangle) S_7 \right\}$$

be the infinite non commutative subset interval polynomial semiring.

Example 2.91: Let S = {Collection of all subsets from the interval polynomial ring

$$M = \left\{ \sum_{i=0}^{12} [a_i, b_i] x^i \;\middle|\; a_i, b_i \in ZS_{12}, 0 \leq i \leq 12, x^{13} = 1 \right\}$$

be an infinite subset interval polynomial non commutative semiring.

Clearly S has subset interval zero divisors which are infinite in number.

S has no nontrivial subset interval idempotents other than A = {[0, 1]} and B = {[1, 0]} in S.

Example 2.92: Let S = {Collection of all subsets from the interval polynomial ring

$$M = \left\{ \sum_{i=0}^{\infty} [a_i b_i] x^i \,\middle|\, a_i, b_i \in \langle Z \cup I \rangle) \; S(3) \right\}\}$$

be the subset interval polynomial semiring. S has infinite number of zero divisors.

It has infinite number of subset interval polynomial subsemirings and subset interval polynomial semiring ideals.

Example 2.93: Let S = {Collection of all subsets from the interval polynomial ring

$$M = \left\{ \sum_{i=0}^{\infty} [a_i b_i] x^i \,\middle|\, a_i, b_i \in (\langle Z \cup I \rangle)(S_3 \times D_{2,7}) \right\}\}$$

be the subset interval polynomial semiring.

S has subset interval zero divisor, subset interval units, subset interval idempotents and subset interval polynomial subsemirings. Further S is a non commutative subset interval polynomial semiring.

Example 2.94: Let S = {Collection of all subsets from the interval polynomial ring

$$M = \left\{ \sum_{i=0}^{3} [a_i b_i] x^i \,\middle|\, x^4 = 1, a_i, b_i \in C(Z_3)\,(g_1, g_2)\; g_1^2 = 0,\; g_2^2 = g_2, \right.$$
$$\left. g_1 g_2 = g_2 g_1 = 0, \leq i \leq 3 \right\}\}$$

be the subset interval polynomial semiring.

S is of finite order commutative has subset interval zero divisors and subset interval units.

However $A_1 = \{[0, g_2]\}$, $A_2 = \{[g_2, g_2]\}$ and $A_3 = \{[g_2, 0]\}$ are some of the interval subset idempotents of S.

$B_1 = \{[g_2, 1]\}$, $B_2 = \{[1, g_2]\}$, $B_3 = \{[0, 1]\}$ and $B_4 = \{[1, 0]\}$ are also interval subset idempotents of S.

Example 2.95: Let S = {Collection of all subsets from the interval polynomial ring

$$M = \left\{ \sum_{i=0}^{3} [a_i, b_i] x^i \;\middle|\; a_i, b_i \in \langle Z_{15} \cup I\rangle\, S_4 \right\}\}$$

be the subset interval polynomial semiring of infinite order which is non commutative.

Now having seen examples of subset interval polynomial semirings of finite and infinite order and subset interval matrix semirings we now propose some problems for the reader.

Problems :

1. Let S = {Collection of all subsets from the ring Z_{24}} be the subset semiring of the ring Z_{24}.

   (i) Find o(S).
   (ii) Find subset zero divisors of S.
   (iii) Find subset idempotents of S.
   (iv) Find subset units of S.

2. Let S_1 = {Collection of all subsets from the ring $R = Z_{12} \times Z_{20}$} be the subset semiring of the ring R.

   Study questions (i) to (iv) of problem 1 for this S_1.

3. Let S_2 = {Collection of all subsets from the ring $R = Z_7(g)$ with $g^2 = 0$ } be the subset semiring.

   Study questions (i) to (iv) of problem 1 for this S_2.

4. Let S_3 = {Collection of all subsets from the neutrosophic ring $\langle Z_6 \cup I\rangle$} be the subset neutrosophic semiring of the neutrosophic ring $\langle Z_6 \cup I\rangle$.

   Study questions (i) to (iv) of problem 1 for this S_3.

5. Let S_4 = {Collection of all subsets from the neutrosophic finite complex modulo integer ring R = C ($\langle Z_{24} \cup I\rangle$)} be the subset finite neutrosophic complex modulo integer ring.

   Study questions (i) to (iv) of problem 1 for this S_4.

6. Let S_5 = {Collection of all subsets from the ring $R = C(Z_{15}) (g_1, g_2)$ where $g_1^2 = 0$, $g_2^2 = g_2$, $g_1g_2 = g_2g_1 = 0$} be the subset finite mixed dual complex semiring of R.

   Study questions (i) to (iv) of problem 1 for this S_5.

7. Let S = {Collection of all subsets from the ring $R = C(Z_{22} \cup I) (g)$, $g^2 = 0$} be the subset semiring.

   Study questions (i) to (iv) of problem 1 for this S.

8. Let S = {Collection of all subsets from the ring Z_{45}} be the subset semiring of Z_{45}.

   (i) Find o(S).
   (ii) Find all subset subsemirings of S.
   (iii) Find all subset semiring ideals of S.
   (iv) Find all subset subsemirings which are not subset semiring ideals of S.

9. Let S_1 = {Collection of all subsets from the ring $Z_{11} \times Z_7$} be the subset semiring.

   Study questions (i) to (iv) of problem (8) for this S_1.

10. Let S_2 = {Collection of all subsets from the ring Z_2S_4 } be the subset semiring of S.

    Study questions (i) to (iv) of problem (8) for this S_2.

11. Let S = {Collection of all subsets from the ring Z_6S_3} be the subset semiring.

    Study questions (i) to (iv) of problem (8) for this S.

12. Let S = {Collection of all subsets from the ring $C(\langle Z_{12} \cup I\rangle)$} be the subset semiring.

    Study questions (i) to (iv) of problem (8) for this S.

13. Let S = {Collection of all subsets from the ring $C(Z_{11})$ (g_1, g_2); $g_1^2 = 0$, $g_2^2 = g_2$, $g_1g_2 = g_2g_1 = 0$} be the subset semiring.

    Study questions (i) to (iv) of problem (8) for this S.

14. Let S = {Collection of all subsets from the ring Z_{40} (g_1, g_2, g_3); $g_1^2 = 0$, $g_2^2 = g_2$, $g_3^2 = -g_3$ with $g_ig_j = g_jg_i = 0$ if $i \neq j$, $1 \leq i, j \leq 3$} be the subset semiring.

    Study question (i) to (iv) of problem (8) for this S.

    (i) Can S have subset units?
    (ii) Can S have Smarandache subset units?
    (iii) Can S have subset idempotents?
    (iv) Can S have subset S-idempotents?

15. Find all subsets semiring ideals of and Smarandache subset semiring ideals of
    S = {Collection of all subsets from the ring Z_{48}}, the subset semiring of the ring Z_{48} of type I.

16. Find all subset subsemirings of
S = {Collection of all subsets from the ring Z_{35}} the subset semiring of Z_{35} which are not subset semiring ideals of S.

17. Find all the subset subsemirings of
S = {Collection of subsets of the ring $Z_{40} \times Z_{28}$}; the subset subsemiring which are not subset semiring ideals of S.

18. Let S = {Collection of all subsets from the ring
$Z_6 \times Z_{12} \times Z_9$} be the subset semiring.

    (i) Find o(S).
    (ii) Find all subset zero divisors of S.
    (iii) Find all subset units of S.
    (iv) Find all subset idempotents of S.
    (v) Find all subset semiring ideals of S.
    (vi) Find all subset subsemirings of S.
    (vii) Find all subset subsemirings which are not subset semiring ideals.

19. Let
S = {Collection of all subsets from the group ring Z_2S_4} be the subset semiring.

    (i) Study questions (i) to (vii) of problem (18) for this S.
    (ii) Prove S is non commutative.
    (iii) Can S have subset semiring right ideals which are not subset semiring left ideals?

20. Let
S = {Collection of all subsets from the group ring Z_5A_4} be the subset semiring of Z_5A_4.

    (i) Prove S is a non commutative subset semiring.
    (ii) Study questions (i) to (vii) of problem (18) for this S.

21. Let
S = {Collection of all subsets from the group ring $ZD_{2,7}$} be the subset semiring of the group ring $ZD_{2,7}$.

    (i) Study questions (i) to (vii) of problem (18) for this S.
    (ii) Can S have subset idempotents?
    (iii) Prove S has subset units.
    (iv) If Z is replaced by Q show S has subset idempotents.
    (v) Any other interesting feature enjoyed S.

22. Give an example of a subset semiring of a ring which has no subset zero divisors.

23. Does there exist a subset semiring of a ring which has no subset units?

24. Give an example of a subset semiring which has no subset idempotents.

25. Is it possible to have a finite order subset semiring which has no subset units.

26. Does there exist a finite subset semiring which has no subset zero divisors?

27. Let S = {Collection of all subsets from the ring $Z_{12}D_{26}$} be the subset semiring.

    (i) Study questions (i) to (vii) of problem (18) for this S.
    (ii) Enlist all subset semiring left ideals which are not subset semiring right ideals.

28. Let S = {Collection of all subsets from the ring $QD_{2,7}$} be the subset semiring of the ring $QD_{2,7}$.

    (i) Can S have right subset semiring ideals which are not left subset semiring ideals?
    (ii) Find all subset semiring ideals of S.

(iii) Can S have subset zero divisors?
(iv) Can S have subset units?
(v) Can S have subset idempotents?
(vi) Can S have subset subsemirings which are not subset semiring ideals?

29. Let
$S = \{$Collection of subsets from the ring $(Z_9 \times Z_{12} \times Z_6)S_3\}$ be the subset semiring.

Study questions (i) to (vii) of problem (18) for this S.

30. Is it ever possible to have a finite subset semiring of a ring to be free from subset zero divisors?

31. Let S = {Collection of subsets from the ring RS(3)} be the subset semiring of the ring RS(3).
Mention or derive all properties associated with is S.

32. If R is replaced by Q in problem 31 study that S.

33. If R is replaced by Z in problem 31 study that S.

34. Let $S = \{$Collection of subsets from the ring $Z_7(S_3 \times A_4)\}$ be the subset semiring.

Study questions (i) to (vii) of problem (18) for this S.

35. Let
$S = \{$Collection of subsets from the ring $Z_2(S_3 \times A_4 \times S_5)\}$ be the subset semiring.

Study questions (i) to (vii) of problem (18) for this S.

36. Let $S = \{$Collection of subsets from the ring $Z_{12}S(8)\}$ be the subset semiring.

Study questions (i) to (vii) of problem (18) for this S.

37. Let S = {Collection of subsets from the ring $(Z_7 \times Z_2 \times S_{12} \times Z_3)$ $(S_3 \times A_4)$} be the subset semiring.

    Study questions (i) to (vii) of problem (18) for this S.

38. Let S = {Collection of subsets from the ring $C(\langle Z_9 \cup I \rangle)$} be the subset semiring.

    Study questions (i) to (vii) of problem (18) for this S.

39. Let S = {Collection of subsets from the ring $C(Z_{18})(g)$ where $g^2 = g$} be the subset semiring.

    Study questions (i) to (vii) of problem (18) for this S.

40. Let S = {Collection of subsets from the ring $C(Z_6) \times C(\langle Z_{19} \cup I \rangle)$} be the subset semiring.

    Study questions (i) to (vii) of problem (18) for this S.

41. Let S = {Collection of subsets from the ring Z_{12} (g_1, g_2, g_3) where $g_1^2 = 0$, $g_2^2 = g_2$, and $g_3^2 = -g_3$ with $g_ig_j = g_jg_i = 0$ if $i \neq j$, $1 \leq i, j \leq 3$} be the subset semiring.

    Study questions (i) to (vii) of problem (18) for this S.

42. Let S = {Collection of subsets from the ring $C(Z_6)$ S_3} be the subset semiring.

    Study questions (i) to (vii) of problem (18) for this S.

43. Let S = {Collection of subsets from the ring $(\langle Z_{10} \cup I \rangle)(g_1,g_2)$ S_3 with $g_1^2 = 0$, $g_2^2 = -g_2$, $g_1g_2 = g_2g_1 = 0$} be the subset semiring of the ring $(\langle Z_{10} \cup I \rangle)$ (g_1, g_2) S_3.

    Study questions (i) to (vii) of problem (18) for this S.

44. Let
$S = \{$Collection of subsets from the ring $C(\langle Z_{15} \cup I\rangle\ (S(4))\}$ be the subset semiring.

Study questions (i) to (vii) of problem (18) for this S.

45. Prove if instead of using rings interval rings are used to from subset semiring show they always contain subset zero divisors.

46. Let $S = \{$Collection of subsets from the ring $M = \{[a, b] \mid a, b \in Z\}\}$ be the subset semiring of the ring of M.

(i) Show S has subset interval zero divisors.
(ii) Is S a Smarandache subset semiring?
(iii) Can S have subset idempotents?
(iv) Find subset semiring ideals of S.

47. Let $S = \{$Collection of subsets from the ring $M = \{[a, b] \mid a, b \in Z_{12}\}\}$ be the subset interval semiring.

(i) Find o(S).
(ii) Find all subset interval zero divisors.
(iii) Find all subsets interval idempotents.
(iv) Find all subset interval units.
(v) Find all subset interval subsemirings which are not subset semiring interval ideals.
(vi) Find all subset interval semiring ideals of S.

48. Let $S = \{$Collection of subsets from the ring $M = \{[a, b] \mid a, b \in \langle Z_{21} \cup I\rangle\}\}$ be the subset interval semiring.

Study questions (i) to (vi) of problem (47) for this S.

49. Let $S = \{$Collection of subsets from the interval ring $M = \{[a, b] \mid a, b \in C(Z_{42})\ (g)$ with $g^2 = 0\}\}$ be the subset interval semiring.

Study questions (i) to (vi) of problem (47) for this S.

50. Let S = {Collection of subsets from the interval ring M = {[a, b] | a, b $\in C(\langle Z_{46} \cup I \rangle)$} be the subset semiring.

    Study questions (i) to (vi) of problem (47) for this S.

51. Let S = {Collection of subsets from the interval ring M = {[a, b] | a, b $\in C(\langle Z_7 \cup I\rangle(g_1)$ with $g_1^2 = 0$} be the subset semiring.

    Study questions (i) to (vi) of problem (47) for this S.

52. Let S = {Collection of subsets from the interval ring M = {[a, b] | a, b $\in (\langle Z_{48} \cup I \rangle) S_4$} be the subset semiring.

    Study questions (i) to (vi) of problem (47) for this S.

53. Let S_1 = {Collection of subsets from the interval ring M = {[a, b] | a, b $\in Z_{24}S_5$}} be the subset interval semiring.

    Study questions (i) to (vi) of problem (47) for this S_1.

54. Find some special features enjoyed by subset interval semirings.

55. Let S = {Collection of subsets from the interval matrix ring

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \\ \vdots & \vdots \\ [a_{19}, b_{19}] & [a_{20}, b_{20}] \end{bmatrix} \right| a_i, b_i \in Z_6; 1 \leq i \leq 20 \right\} \}$$

    be the subset semiring of the interval matrix ring M.

    (i) Find o(S).
    (ii) Find subset interval idempotents of S.

(iii) Find subset interval units of S.
(iv) Find all S-subset interval idempotents of S.
(v) Find all subset interval matrix semiring ideals of S.
(vi) Find all subsets interval matrix semirings of S which matrix semirings ideals of S.

56. Let S = {Collection of all susbets form interval matrix ring

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] & \ldots & [a_{12}, b_{12}] \\ [a_{13}, b_{13}] & [a_{14}, b_{14}] & \ldots & [a_{24}, b_{24}] \\ [a_{25}, b_{25}] & [a_{26}, b_{26}] & \ldots & [a_{36}, b_{36}] \end{bmatrix} \middle| \; a_i, b_i \in Z_{23};\ 1 \leq i \leq 36 \right\} \}$$

be the subset interval matrix semiring.

Study questions (i) to (vi) of problem (55) for this S.

57. Let S = {Collection of all susbets form interval matrix ring

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] & \ldots & [a_6, b_6] \\ [a_7, b_7] & [a_8, b_8] & \ldots & [a_{12}, b_{12}] \\ \vdots & \vdots & & \vdots \\ [a_{31}, b_{31}] & [a_{32}, b_{32}] & \ldots & [a_{36}, b_{36}] \end{bmatrix} \middle| \; a_i, b_i \in Z_{24};\ 1 \leq i \leq 36 \right\} \}$$

be the subset interval matrix semiring.

Study questions (i) to (vi) of problem (55) for this S.

58. Let S = {Collection of all subsets form interval matrix ring M = {([a_1, b_1], [a_2, b_2], …, [a_{17}, b_{17}]) | $a_i, b_i \in C(Z_{20})$; $1 \leq i \leq 17$}} be the subset interval matrix semiring.

Study questions (i) to (vi) of problem (55) for this S.

59. Let S = {Collection of subsets from the ring

$$M=\left\{\left.\begin{bmatrix} [a_1,b_1] & [a_2,b_2] \\ [a_3,b_3] & [a_4,b_4] \\ \vdots & \vdots \\ [a_{12},b_{12}] & [a_{24},b_{24}] \end{bmatrix}\right| a_i,b_i \in C(\langle Z_{23} \cup I\rangle);\ 1 \leq i \leq 24\}\right\}$$

be the subset interval matrix semiring.

Study questions (i) to (vi) of problem (55) for this S.

60. Let S = {Collection of all susbets form interval matrix ring

$$M = \left\{\left.\begin{bmatrix} [a_1,b_1] & [a_2,b_2] & \ldots & [a_{19},b_{19}] \\ [a_7,b_7] & [a_8,b_8] & \ldots & [a_{38},b_{38}] \end{bmatrix}\right| a_i,\ b_i \in\right.$$

$Z_{15}(g_1, g_2, g_3)$, $g_1^2 = 0$, $g_2^2 = g_2$ and $g_3^2 = -g_3$, $1 \leq i \leq 38\}\}$

be the subset interval matrix semiring.

Study questions (i) to (vi) of problem (55) for this S.

61. Let S = {Collection of all subsets form interval matrix ring

$$M = \left\{\left.\begin{bmatrix} [a_1,b_1] & [a_2,b_2] & [a_3,b_3] \\ [a_4,b_4] & [a_5,b_5] & [a_6,b_6] \\ [a_7,b_7] & [a_8,b_8] & [a_9,b_9] \end{bmatrix}\right| a_i,\ b_i \in C(\langle Z_{42} \cup I\rangle\right.$$

(g_1, g_2); $g_1^2 = 0$, $g_2^2 = g_2$, $g_1g_2 = g_2g_1 = 0$, $1 \leq i \leq 9\}$

be the subset interval matrix semiring of M.

Study questions (i) to (vi) of problem (55) for this S.

Is S a Smarandache subset matrix interval semiring?

Find some special features enjoyed by subset interval matrix semiring of infinite order.

62. Let S = {Collection of all susbets form interval matrix ring

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] & [a_3, b_3] \\ [a_4, b_4] & [a_5, b_5] & [a_6, b_6] \\ \vdots & \vdots & \vdots \\ [a_{28}, b_{28}] & [a_{29}, b_{29}] & [a_{30}, b_{30}] \end{bmatrix} \middle| \; a_i, b_i \in Z_{10}A_4; \; 1 \le i \le 30 \right\}$$

be the subset interval matrix semiring.

(i) Study questions (i) to (vi) of problem (55) for this S .
(ii) Prove S is non commutative.
(iii) Is S a Smarandache subset interval matrix semiring.

63. Let $S = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ [a_4, b_4] & [a_5, b_5] \\ \vdots & \vdots \\ [a_{19}, b_{19}] & [a_{20}, b_{20}] \end{bmatrix} \middle| \; a_i, b_i \in Z_{12}S_7; \; 1 \le i \le 20 \right\}$

be the subset interval matrix semiring.

(i) Find o(S).
(ii) Find all subset interval zero divisors.
(iii) Find all subset interval idempotents.
(iv) Find all subset interval units.
(v) Find all subset interval matrix semiring ideals.
(vi) Find all subset interval matrix subsemirings which are not ideals.

64. Let S = {Collection of all susbets form interval matrix ring

$$M = \left\{ \left. \begin{bmatrix} [a_1,b_1] & [a_2,b_2] & \ldots & [a_{10},b_{10}] \\ [a_7,b_7] & [a_8,b_8] & \ldots & [a_{20},b_{20}] \end{bmatrix} \right| a_i,\, b_i \in \right.$$

$$C(\langle Z_{12} \cup I \rangle)\ (S(3)),\ 1 \le i \le 20\}\}$$

be the subset interval matrix semiring.

Study questions (i) to (vi) of problem (55) for this S.

65. Let S = {Collection of all subsets form interval matrix ring

$$M = \left\{ \left. \begin{bmatrix} [a_1,b_1] & [a_2,b_2] & [a_3,b_3] \\ [a_4,b_4] & [a_5,b_5] & [a_6,b_6] \\ [a_7,b_7] & [a_8,b_8] & [a_9,b_9] \end{bmatrix} \right| a_i,\, b_i \in C(\langle Z_4 \cup I \rangle \right.$$

$$(S(4)),\ 1 \le i \le 8\}$$

be the subset interval matrix semiring of finite order.

Study questions (i) to (vi) of problem (55) for this S.

66. Let S = {Collection of subsets from the interval ring

$$M = \left\{ \left. \begin{bmatrix} [a_1,b_1] \\ [a_2,b_2] \\ \vdots \\ [a_{10},b_{10}] \end{bmatrix} \right| a_i,\, b_i \in [Z_{40} \times C(Z_{24})]\ S_7;\ 1 \le i \le 10\}\} \right.$$

be the subset interval matrix semiring.

Study questions (i) to (vi) of problem (55) for this S.

67. Let S = {Collection of subsets from the ring

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] & \ldots & [a_9, b_9] \\ [a_{10}, b_{10}] & \ldots & [a_{18}, b_{18}] \\ [a_{19}, b_{19}] & \ldots & [a_{27}, b_{27}] \end{bmatrix} \right| a_i, b_i \in [Z_{40} \times Z_{10} \times C(Z_3)]\ A_5 \right\}\}$$

be the subset interval matrix semiring of M.

Study questions (i) to (vi) of problem (55) for this S.

68. Let S = {Collection of subsets from the interval polynomial ring

$$M = \left\{ \left. \sum_{i=0}^{\infty} [a_i, b_i] x^i \,\right|\, a_i, b_i \in \langle Z_{10} \cup I \rangle \right\}\}$$

be the subset interval matrix semiring of M.

Study questions (i) to (vi) of problem (55) for this S .

69. Let S = {Collection of subsets from the interval polynomial ring

$$M = \left\{ \left. \sum_{i=0}^{10} [a_i, b_i] x^i \,\right|\, a_i, b_i \in \langle Z_8 \cup I \rangle,\ 0 \leq i \leq 10,\ x^{11} = 1 \right\}\}$$

be the subset interval matrix semiring.

Study questions (i) to (vi) of problem (55) for this S.

70. Let S = {Collection of subsets from the interval polynomial ring

$$M = \left\{ \sum_{i=0}^{6} [a_i, b_i] x^i \;\middle|\; a_i, b_i \in C(Z_7);\ 0 \le i \le 6,\ x^7 = 1 \right\}$$

be the subset interval matrix semiring.

Study questions (i) to (vi) of problem (55) for this S .

71. Let S = {Collection of subsets from the interval polynomial ring

$$M = \left\{ \sum_{i=0}^{7} [a_i, b_i] x^i \;\middle|\; a_i, b_i \in Z_{36},\ x^8 = 1,\ 0 \le i \le 7 \right\}\}$$

be the subset interval matrix semiring.

Study questions (i) to (vi) of problem (55) for this S.

Chapter Three

SUBSET SEMIRINGS OF TYPE II

In this chapter for the first time authors introduce the notion of subset semirings of type two where we use subsets from the semiring. Here we describe and develop these concepts. We give both commutative and non commutative, finite and infinite subset semirings of type II.

These semirings also contain subset units, subset zero divisors and subset idempotents.

DEFINITION 3.1: *Let S = {Collection of all subsets from a semiring R}. S under the operations of the semiring R is a semiring defined as the subset semiring of type II.*

Here we describe this situation by some examples.

Example 3.1: Let S = {Collection of all subsets from the semiring $Z^+ \cup \{0\}$} be the subset semiring of type II.

For take A = {3, 4, 8, 12} and B = {0, 1, 5, 7} $\in$ S.

$$A + B = \{3, 4, 8, 12\} + \{0, 1, 5, 7\}$$

$= \{3, 4, 8, 12, 5, 9, 15, 21, 13, 17, 10, 11\} \in S.$

$A \times B$

$= \{3, 4, 8, 12\} \times \{0, 1, 5, 7\}$

$= \{3 \times 0, 4 \times 0, 8 \times 0, 12 \times 0, 3 \times 1, 4 \times 1, 8 \times 1, 12 \times 1, 3 \times 5, 4 \times 5, 8 \times 5, 12 \times 5, 12 \times 7, 4 \times 7, 3 \times 7, 8 \times 7\}$

$= \{0, 3, 4, 8, 12, 15, 20, 40, 60, 84, 28, 21, 56\}.$

This is the way operations on S are performed. S is of infinite order. S is a commutative subset semiring of type II.

Example 3.2: Let
S = {Collection of all subsets from the semiring $\langle Z^+ \cup \{0\} \cup I\rangle$} be the subset semiring of type II.

Take A = {4I, 3I + 2, 4 + 5I, 7I + 1} and
B = {I, 2+5I, 7+6I} $\in$ S.

$A + B$

$= \{4I, 3I+2, 4+5I, 7I+1\} + \{I, 2+5I, 7+6I\}$

= {5I, 4I+2, 6I+4m 8I+1, 2+9I, 4+8I, 6+10I, 3 + 12I, 7+10I, 9I+9, 11I+11, 8+13I} $\in$ S.

Now

$A \times B$

$= \{4I, 3I + 2, 4 + 5I, 7I + 1\} \times [I, 2+5I, 7+6I\}$

$= \{4I \times I, 3I+2 \times I, 4 + 5I \times I, 7I + 1 \times I, 4I \times 2 + 5I, 3I + 2 \times 2 + 5I, 4 + 5I \times 2 + 3I, 7I + 1 \times 3I + 2, 4I \times 7 + 6I, 3I + 2 \times 7 + 6I, 4 + 5I \times 7 + 6I, 7I + 1 \times 7 + 6I\}$

$= \{4I, 5I, 9I, 8I, 28I, 4 + 31I, 8 + 37I, 2 + 38I, 52I, 14 + 51I, 28 + 89I, 7 + 97I\} \in S.$

This is the way operations are performed on S.

S will also be known as the subset neutrosophic semiring of type II.

Example 3.3: Let S = {Collection of all subsets from the semiring $(Z^+ \cup \{0\})(g) = \{a + bg \mid a, b \in Z^+ \cup \{0\}, g^2 = 0\}$} be the subset semiring of infinite order which is commutative. S has subset zero divisors.

For if $A = \{5g, 3g, 2g, 28g, 40g, 55g\}$ and
$B = \{19g, 28g, 56g, g\} \in S$.

We see $A \times B = \{5g, 3g, 2g, 28g, 40g, 55g\} \times \{19g, 28g, 56g, g\} = \{0\}$.

This S has infinite number of subset zero divisors.

Example 3.4: Let
S = {Collection of all subsets from group semiring $(Z^+ \cup \{0\})S_3$} be the subset semiring of type II. S has subset units.

$$\text{For take } A = \left\{\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}\right\} \in S.$$

$$\text{We see } A \times A = \left\{\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}\right\} = \{1\};$$

thus S has subset units. S is a non commutative subset semiring of type II.

S is of infinite order.

$$\text{Take } A = \left\{\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}\right\} \text{ and } B = \left\{\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}\right\} \in S.$$

$$A \times B = \left\{\begin{pmatrix}1 & 2 & 3\\1 & 3 & 2\end{pmatrix}\right\} \times \left\{\begin{pmatrix}1 & 2 & 3\\2 & 1 & 3\end{pmatrix}\right\}$$

$$= \left\{\begin{pmatrix}1 & 2 & 3\\1 & 3 & 2\end{pmatrix} \times \begin{pmatrix}1 & 2 & 3\\2 & 1 & 3\end{pmatrix}\right\}$$

$$= \left\{\begin{pmatrix}1 & 2 & 3\\2 & 3 & 1\end{pmatrix}\right\} \qquad \ldots \text{ I}$$

Consider

$$B \times A = \left\{\begin{pmatrix}1 & 2 & 3\\2 & 1 & 3\end{pmatrix}\right\} \times \left\{\begin{pmatrix}1 & 2 & 3\\1 & 3 & 2\end{pmatrix}\right\}$$

$$= \left\{\begin{pmatrix}1 & 2 & 3\\2 & 1 & 3\end{pmatrix} \times \begin{pmatrix}1 & 2 & 3\\1 & 3 & 2\end{pmatrix}\right\}$$

$$= \left\{\begin{pmatrix}1 & 2 & 3\\3 & 1 & 2\end{pmatrix}\right\} \qquad \ldots \text{ II}$$

Clearly I and II are distinct so $A \times B \neq B \times A$; thus S is a non commutative subset semiring of infinite order.

Example 3.5: Let
S = {Collection of all subsets from group semiring $(Q^+ \cup \{0\})S_4$} be the subset semiring of infinite order. S is non commutative of infinite order. S has subset units and subset idempotents.

$A = \left\{\frac{1}{2}\right\}$ and $B = \{2\} \in S$ is a subset unit as $A \times B = \{1\}$.

$$\text{Take } A = \left\{\frac{1}{2}\left(1 + \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 3 & 4 \end{pmatrix}\right)\right\} \in S$$

$$A \times A = \left\{\frac{1}{2}\left(1 + \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 3 & 4 \end{pmatrix}\right) \times \frac{1}{2}\left(1 + \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 3 & 4 \end{pmatrix}\right)\right\}$$

$$= \left\{\frac{1}{2\times 2}\left(1 + \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 3 & 4 \end{pmatrix}\right)^2\right\}$$

$$= \left\{\frac{1}{4}\left(1 + 1 + 2\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 3 & 4 \end{pmatrix}\right)\right\}$$

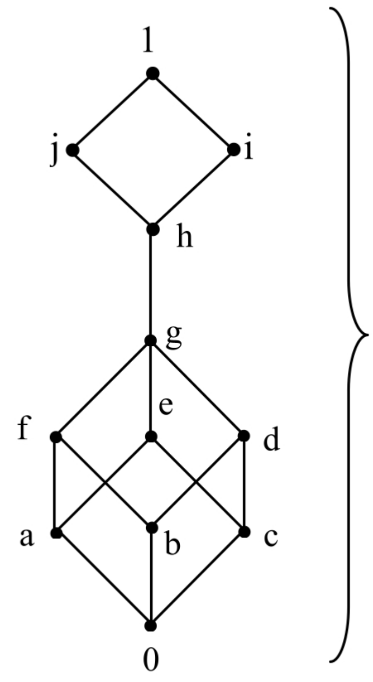
$$= \left\{\frac{1}{2}\left(1 + \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 3 & 4 \end{pmatrix}\right)\right\} = A \in S.$$

Thus S has subset idempotents.

Example 3.6: Let S = {Collection of all subsets from the semiring $(Z^+ \cup \{0\})S_7$} be the subset semiring of type II. Infact S is of infinite order non commutative has only subset units.

Example 3.7: Let

S = {Collection of all subsets from the semiring L =



be the subset semiring of type II.

S has subset idempotents and subset zero divisors. S is commutative and is of finite order.

Take A = {a} and B = {b} ∈ S with

$$A \times B = \{a\} \times \{b\} = \{0\}.$$

A is a subset zero divisor of S.

Let A = {a, b, 0} ∈ S.

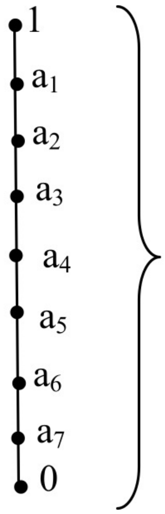
$$\begin{aligned} A \times A &= \{a, b, 0\} \times \{0, a, b\} \\ &= \{a, b, 0\} = A \in S. \end{aligned}$$

A is a subset idempotent of S. B = {a} ∈ S is also a subset idempotent.

Every singleton element in L is a subset idempotent of S; that is if x ∈ L then X = {x} ∈ S is such that X × X = {x} × {x} = {x} = X ∈ S, is a subset idempotent of S.

Example 3.8: Let S = {Collection of all subsets from the lattice

L =



be the subset semiring of L. S is of finite order.

S has no subset zero divisors. S has subset idempotents and all these subset idempotent sets are of maximum cardinality nine.

If A = $\{0, 1, a_i\} \in S$; $a_i \in L$; $1 \leq i \leq 7$ is such that $A \times A = \{0, 1, a_i\} \times \{0, 1, a_i\} = \{0, 1, a_i\} = A \in S$ is a subset idempotent of S.

$B = \{1, a_i\}$ and $C = \{0, a_i\} \in S$ are also subset idempotents of S.

Take $D_i = \{a_i\} \in S$ is such that $D_i \times D_i = \{a_i\} \times \{a_i\} = \{a_i\}$; $a_i \in L$, $1 \leq i \leq 7$ are also subset idempotents of S.

However S has no subset units.

Take $A = \{a_1, a_3, 0\}$ and $B = \{a_5, a_6, 1\} \in S$.

$$\begin{aligned} A \cup B &= \{a_1 \cup a_5, a_1 \cup a_6, a_1 \cup 1, a_3 \cup a_5, a_3 \cup a_6, a_3 \cup 1, \\ &\quad 0 \cup a_5, 0 \cup a_6, 0 \cup 1\} \\ &= \{a_1, 1, a_3, a_6, a_5\} \in S. \end{aligned}$$

$A \cap B$

$= \{a_1, a_3, 0\} \cap \{1, a_6, a_5\}$

$= \{a_1 \cap 1, a_3 \cap 1, 0 \cap 1, a_1 \cap a_6, a_3 \cap a_6, 0 \cap a_6,$
$0 \cap a_5, 0 \cap a_6, 0 \cap 1\}$

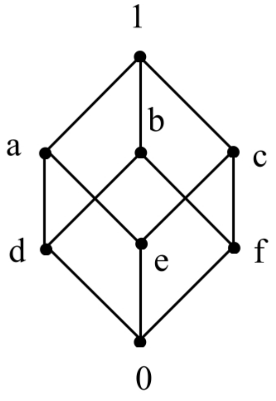
$= \{a_1, a_3, a_5, 0, a_6\} \in S.$

This is the way operations on S are performed.

Clearly
$A \cap B = B \cap A$ and $A \cup B = B \cup A$ for all $A, B \in S$.

S has only subset idempotents and no subset zero divisors.

Example 3.9: Let S = {Collection of all subsets from the semiring



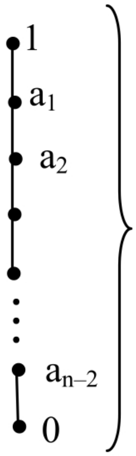
;

which is a Boolean algebra of order 8} be the subset semiring of type II. S has subset idempotents and subset zero divisors.

In view of all these we have the following theorem.

THEOREM 3.1: *Let S = {Collection of all subsets from the semiring which is a chain lattice;*

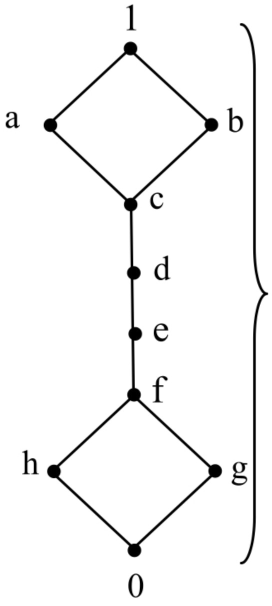
$C_n =$



be the subset semiring of type II. S has no subset zero divisors or subset units only subset idempotents.

Proof is direct and hence left as an exercise to the reader.

Example 3.10: Let S = {Collection of all subsets from the semiring L =



be the subset semiring of finite order.

S has subset zero divisors and subset units. Infact L is not a chain lattice only a distributive lattice.

THEOREM 3.2: *Let S = {Collection of all subsets from the Boolean algebra with* $2^{|X|}$ *elements} be the subset semiring. S has subset idempotents and subset zero divisors and* $o(S) = 2^{2^{|X|}}$.

Proof is direct and hence left as an exercise to the reader.

Now we proceed onto give more examples of subset semirings using lattices of type II.

Example 3.11: Let
S = {Collection of all subsets from the semiring $\langle Q^{+} \cup I \cup \{0\}\rangle$} be the subset semiring of type II. This is of infinite order.

S has subset zero divisors, subset units and subset idempotents. A = {n} where $n \in Q^{+}$, there exist a unique $m \in Q^{+}$ with B = {m} ∈ S such that A × B = {n} × {m} = {1} is a subset unit of S.

Infact only singletons can be subset units and we have infinite number of subset units.

The subset idempotents are {0}, {1, 0}, {1}, {I}, {I, 0}, {1, I}, {0, 1, I} are the subset idempotents.

However S has no subset zero divisors.

Example 3.12: Let S = {Collection of all subsets from the dual number semiring $(Q^{+} \cup \{0\})(g)$ where $g^2 = 0$} be the subset semiring.

S has infinite number of subset zero divisors.

A = {3g, 5g, 2g, 10g, 11/3g, 45/17g} and

B = {12g, 3/11g, 10/7g, 40/19g} ∈ S.

A × B = {3g, 5g, 2g, 10g, 11/g, 45g/17} × {12g, 3/11g, 10/7g, 40/19g} = {0}.

We see S does not contain infinite number of subset idempotents only a very few subset idempotents.

Example 3.13: Let S = {Collection of all subsets from the special dual like number semiring $\langle Z^+ \cup \{0\} \cup \{g\}\rangle = \{a + bg \mid a, b \in Z^+ \cup \{0\}\}$ with $g^2 = g$} be the subset semiring of type II. S has few subset idempotents no subset zero divisors and subset units.

Example 3.14: Let S = {Collection of all subsets from the special dual like number rational semiring $\langle Q^+ \cup \{0\} \cup \{g\}\rangle = \{a + bg \mid a, b \in Q^+ \cup \{0\};\ g^2 = g\}$} be the subset semiring of infinite order. S has no subset zero divisors only a few subset idempotents and infinite number of subset units.

All these examples of subset semirings are commutative.

Now we proceed onto describe with examples, subset semirings of type II which are non commutative.

Example 3.15: Let S = {Collection of all subsets from the group semiring $(Z^+ \cup \{0\})D_{2.9}$} be the subset semiring. S has subset units. S is non commutative and is of infinite order.

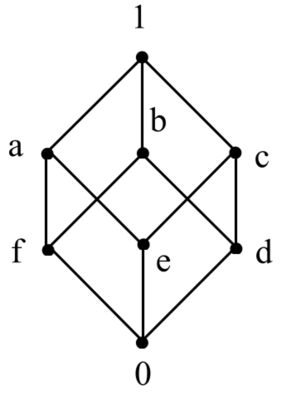
S has finite number of subset units and no subset idempotents other than {0}, {1} and {0, 1}.

Example 3.16: Let

S = {Collection of all subsets from group semiring $(Q^+ \cup \{0\})S_7$} be the subset semiring. S has infinite number of units. S is a non commutative infinite subset semiring.

S has only finite number of subset idempotents S has finite number of subset zero divisors.

Example 3.17: Let S = {Collection of all subsets from the semigroup semiring LS(3) where L =

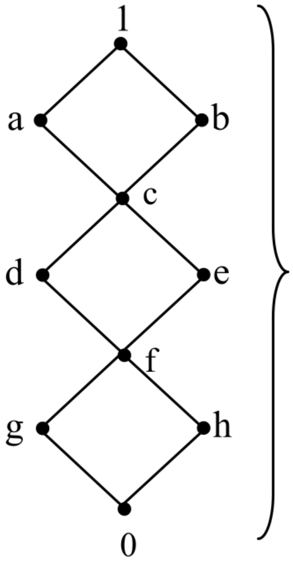


and S(3) is the symmetric semigroup} be the subset semiring of finite order.

S is non commutative subset semiring of type II.

We see S has subset idempotents, subset units and subset zero divisors.

Example 3.18: Let S = {Collection of all subsets from the group semiring LS_3 where L is a lattice given by



be the subset semiring of finite order which is non commutative; we see S has subset idempotents, and subset zero divisors.

Let $A = \{ap_1 + bp_2 + c, gp_3 + hp_4 + d\}$ and
$B = \{cp_3 + ep_5 + 1\} \in S$.

We find

$$\begin{aligned} A + B &= A \cup B \\ &= \{ap_1 + bp_2 + c, gp_3 + hp_4 + d\} \cup \{cp_3 + ep_5 + 1\} \\ &= \{ap_1 + bp_2 + c, gp_3 + hp_4 + d\} + \{cp_3 + cp_5 + 1\} \\ &= \{ap_1 + bp_2 + c + cp_3 + ep_5 + 1, gp_3 + hp_4 + d + \\ &\quad cp_3 + cp_5 + 1\} \\ &= ap_1 + bp_2 + cp_3 + ep_5 + 1, cp_3 + hp_4 + cp_5 + 1\} \\ &\qquad \in S. \end{aligned}$$

We find

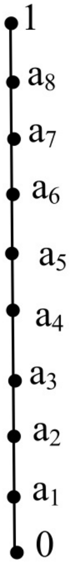
$$\begin{aligned} A \cap B &= A \times B \\ &= \{ap_1 + bp_2 + c, gp_3 + hp_5 + d\} \times \{cp_3 + ep_5 + 1\} \\ &= \{(ap_1 + bp_2 + c) \cap (cp_3 + ep_5 + 1), \\ &\quad (gp_3 + hp_4 + d) \cap (cp_3 + fp_5 + 1)\} \\ &= \{(a \cap c)\, p_1p_3 + (b \cap c)p_2p_3 + (c \cap c)\, p_3 + (a \cap e)p_1p_5 \\ &\quad + (b \cap e)\, p_2p_5 + (c \cap e)\, p_5 + (a \cap 1)p_1 + (b \cap 1)p_2 + \\ &\quad (c \cap 1), (g \cap c)\, p_3 \times p_3 + (h \cap c)\, p_4p_3 + (d \cap c)p_3 + \\ &\quad (g \cap f)p_3\, p_5 + (h \cap f)p_4p_5 + (d \cap f)p_5 + 3 + hp_4 + d\} \\ &= \{cp_4 + cp_5 + cp_3 + ep_2 + ep_3 + ep_5 + ap_1 + bp_2 + c, \\ &\quad gp_3 + hp_4 + d + fp_5 + dp_3 + g + hp_1 + gp_1 + h\} \\ &= \{cp_4 + cp_5 + cp_3 + bp_2 + ap_1 + c, d + fp_1 + dp_3 + \\ &\quad hp_4 + fp_5\} \in S. \end{aligned}$$

This is the way operation ∩ (i.e., ×) and ∪ (i.e., +) are performed in S.

It is easily verified S is a non commutative subset semiring.

A = {a}, B = {b}, C = {c}, D = {d}, E = {e} and so on are subset idempotents of S.

Example 3.19: Let S = {Collection of all subsets from the group semiring $LD_{2,4}$ where $D_{2,4} = \{a,b \mid a^2 = b^4 = 1, bab = a\}$} be the subset semiring of finite order which is non commutative and the lattice L is given by the L =



S has subset idempotents.

Take A = {1 + a} ∈ S.

$$\begin{aligned} A \times A &= \{(1 + a)\} \times \{(1 + a)\} \\ &= \{1 + a + a + a^2\} \\ &= \{1 + a + 1\} \\ &= \{1 + a\} = A. \end{aligned}$$

Let $A_1 = \{1 + b^2\} \in S$.

We see $A_1 \times A_1 = A_1$ so A_1 is also a subset idempotent of S.

We see S has no subset zero divisors.

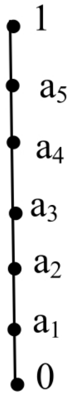
Example 3.20: Let S = {Collection of all subsets from the group semiring $(Z^+ \cup \{0\})$ $(S_3 \times A_4)$} be the subset semiring. S is non commutative and of infinite order.

S has subset units. Only A = {0}, B = {1} and C = {0, 1} are the trivial subset idempotents of S.

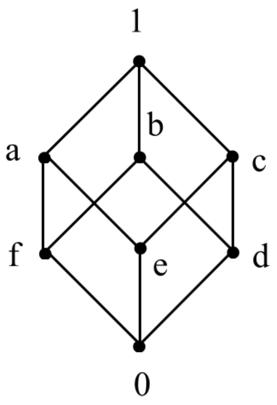
Example 3.21: Let S = {Collection of all subsets from the semigroup semiring $(Q^+ \cup \{0\})$ $(S(3) \times (Z_{12} \times))$} be the subset semiring. S is of infinite order and has infinite number of subset units.

Example 3.22: Let S = {Collection of all subsets from group semiring $(L_1 \times L_2)$ $D_{2,21}$;

where L_1 =



and L_2 =



}

be the subset semiring.

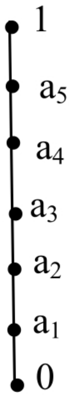
S is of finite order but S is non commutative has subset idempotents given by

$A_1 = \{(1, 1)\}$, $A_2 = \{(1, a)\}$,…, $A_t = \{(1, d)\}$,
$A_{t+1} = \{(a_i, 1)\}$, $A_{t+2} = \{(a_i, a)\}$ and so on.

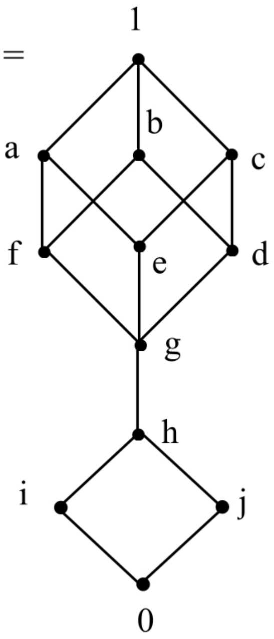
S has subset zero divisors.

Example 3.23: Let S = {Collection of all subsets from the semiring $L_1 \times L_2 \times L_3$ where

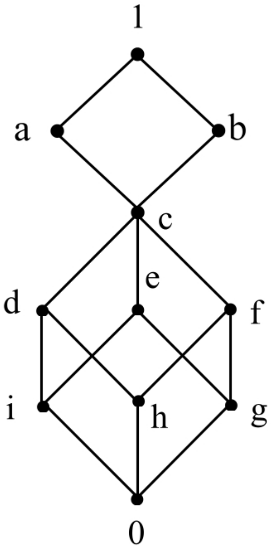
L_1 =



L_2 =



and L_3 =

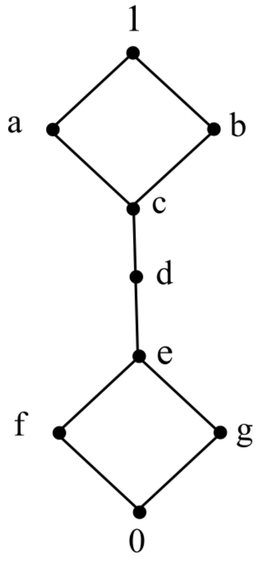


}

be the subset semiring of finite order.

S has subset idempotents and subset zero divisors.

Example 3.24: Let S = {Collection of all subsets from the group semiring $(L_1 \times L_2)$ $(S_3 \times S_4)$ where L_1 =



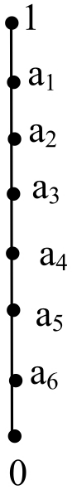
and L_2, a Boolean algebra of order 16} be the subset semiring of finite order which is non commutative and has subset subsemirings, subset idempotents and subset zero divisors.

Example 3.25: Let
S = {Collection of all subsets from the semiring $Z^+ \cup \{0\}$} be the subset semiring of infinite order. This has infinite number of subset subsemiring given by P_n = {Collection of all subsets from the subsemiring $nZ^+ \cup \{0\}$} n = 2, 3, …, ∞. Clearly these subset semirings are also subset ideals of S. S has no nontrivial subset units.

Example 3.26: Let
S = {Collection of all subsets from the semiring $Q^+ \cup \{0\}$} be the subset semiring of infinite order. S has infinite number of subset subsemiring but has no subset semiideals.

S has infinite number of subset units and has only {0}, {1} and {0,1} to be subset idempotents.

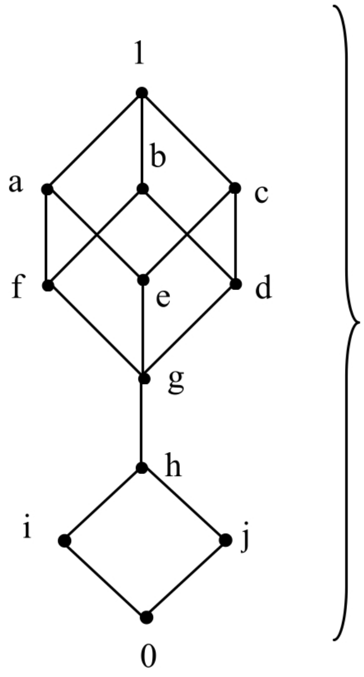
Example 3.27: Let S = {Collection of all subsets from the semiring L =



be the subset semiring. S has subset idempotents. S has subset subsemirings. S has subset ideals.

Example 3.28: Let S = {Collection of all subsets from the semiring L = a Boolean algebra of order thirty two} be the subset semiring. S has subset zero divisors. S has subset idempotents.

Example 3.29: Let S = {Collection of all subsets from the semiring L =



be the subset semiring. S has subset subsemirings, subset ideals, subset idempotents and subset zero divisors.

Now we study more about subset semirings using matrix semirings.

Suppose we have M = {Collection of all m × n matrices with entries from a semiring R} be a matrix semiring.

Let S = {Collection of all subsets from the matrix semiring M}; we define S to be a subset matrix semiring.

Example 3.30: Let S = {Collection of all subsets from the row matrix semiring $M = \{(a_1, \ldots, a_n) \mid a_i \in Z^+ \cup \{0\}, 1 \leq i \leq n\}$} be the subset row matrix semiring.

Clearly this subset row matrix semiring has subset zero divisors and subset idempotents.

However we do not have subset row matrix units. For if A = {(1 1 1 … 1)} is the subset row matrix unit.
B = {(1 0 0 …1 1 1)} ∈ S is a row matrix subset idempotent.

Example 3.31: Let S = {Collection of all subsets from the row matrix semiring $M = \{(a_1\ a_2\ a_3\ a_4\ a_5) \mid a_i \in Q^+ \cup \{0\}, 1 \leq i \leq 5\}$} be the subset row matrix semiring. S has row matrix subset idempotents.

For take A = {(0 0 1 1 1), (0 0 0 0 0), (1 1 0 0 0)} ∈ S;

we see A × A

= {(0 0 1 1 1), (0 0 0 0 0), (1 1 0 0 0)} ×
{(0 0 1 1 1), (0 0 0 0 0), (1 1 0 0 0)}

= {(0 0 1 1 1) × (0 0 1 1 1), (1 1 0 0 0) × (1 1 0 0 0),
(0 0 1 1 1) × (1 1 0 0 0), (1 1 0 0 0) × (0 0 0 1 1 1),
(1 1 0 0 0) × (0 0 0 0 0), (0 0 1 1 1) × (0 0 0 0 0),
(0 0 0 0 0) × (0 0 0 0 0)}

$= \{(1\ 1\ 0\ 0\ 0), (0\ 0\ 1\ 1\ 1), (0\ 0\ 0\ 0\ 0)\}$

$= A.$

Thus A is a subset row matrix idempotent of S. S has several subset row matrix idempotents.

Let $A = \{(1/3, 5/7, 2, 9, 1/17)\}$ and
$B = \{(3, 7/5, 1/2, 1/9, 17)\} \in S$.

We see $A \times B$

$= \{(1/3, 5/7, 2, 9, 1/17)\} \times \{(3, 7/5, 1/2, 1/9, 17)\}$
$= \{(1\ 1\ 1\ 1\ 1)\}$ is the subset unit of S.

Take $A = \{(a_1\ 0\ 0\ 0\ 0)\} \in S$.

All subsets from T = {Collection of all subsets from the collection of matrices $M = \{(0,b_1,b_2,b_3,b_4)$ where $b_i \in Q^+ \cup \{0\}$, $1 \leq i \leq 4\} \subseteq S$ is such that for every $B \in T$ and

$A = \{(a_1\ 0\ 0\ 0\ 0), (a_2\ 0\ 0\ 0\ 0), \ldots, (a_t\ 0\ 0\ 0\ 0)\} \in S$

We have $A \times B = \{(0\ 0\ 0\ 0\ 0)\}$.

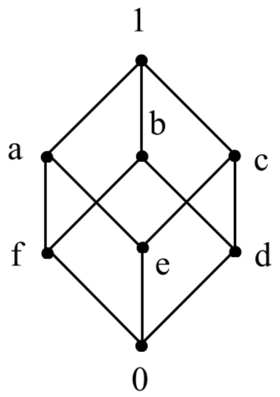
Thus S has infinite number of subset zero divisors.

S has also subset matrix subsemirings which are not subset matrix ideals.

Consider T = {Collection of all subset from the matrix subsemiring $N = \{(a_1, a_2, a_3, a_4, a_5)$ where $a_i \in Z^+ \cup \{0\}$, $1 \leq i \leq 5\}\} \subseteq S$, T is only a subset matrix subsemiring which is not a subset matrix semiring ideal of S.

Inview of all these we have several results which will be enumerated later.

Example 3.32: Let S = {Collection of all subsets from the row matrix semiring $M = \{(a_1\ a_2\ a_3\ a_4\ a_5\ a_6) \mid a_i \in L =$



1 ≤ i ≤ 6}} be the subset row matrix semiring.

S has subset zero divisors and has no subset units. S has subset idempotents. S has subset row matrix subsemirings as well as subset row matrix semiring ideals.

For if A = {(0 0 1 a 1 b)} and
B = {(a b c 1 0 1), (0 1 f e d 0)} ∈ S.

A × B
= A ∩ B
= {(0 0 1 a 1 b)} × {(a b c 1 0 1), (0 1 f e d 0)}

= {(0 0 1 a 1 b) ×(a b c 1 0 1), (0 0 1 a 1 b) × (0 1 f e d 0)}

= {(0 0 c a 0 b) (0 0 f e d 0)} ∈ S.

A ∪ B
= A + B
= {(0 0 1 a 1 b)} ∪ {(a b c 1 0 1), (0 1 f e d 0)}

= {(0 0 1 a 1 b) ∪ (a b c 1 0 1), (0 0 1 a 1 b) ∪ (0 1 f e d 0)}

= {(a b 1 1 1 1), (0 1 1 a 1 b)} ∈ S.

This is the way ∩ and ∪ operations are done on S.

Example 3.33: Let S = {Collection of all subsets from the row matrix group semiring M = $\{(a_1\ a_2\ a_3) \mid a_i \in (Z^+ \cup \{0\})D_{2,5}; 1 \leq i \leq 3\}\}$ be the subset row matrix semiring of M.

S is a non commutative subset semiring.

Let A = $\{(4 + 2a + b^3, 3a + 5b^2, 10b^3 + 5a + 7)\}$ and B = $\{(2a + 1, b, b^4), (7b2+1, 2b+3, a)\} \in S$.

We find

A × B

$= \{(4 + 2a + b^3, 3a + 5b^2, 10b^3 + 5a + 7) \times (2a + 1, b, b^4), (4 + 2a + b^3, 3a + 5b^2, 10b^3 + 5a + 7) \times (7b^2 + 1, 2b + 3, a)\}$

$= \{(4 + 2a + b^3 \times 2a + 1, 3a + 5b^2 \times b, 10b^3 + 5a + 7 \times b^4), (4 + 2a + b^3 \times 7b^2 + 1, 3a + 5b^2 \times 2b + 3, 10b^3 + 5a + 7 \times a)\}$

$= \{(8a + 4a^2 + 2b^3a + 4 + 2ab + b^3, 3ab + 5b^3, 10b^7 + 5ab^4 + 7b^4), (28b^2 + 14ab^2 + 7b^5 + 4 + 2a + b^3, 6ab + 10b^3 + 9a + 15b^2, 10b^3a + 5a^2 + 7a)\}$

$= \{(8a + 8 + 2b^3a + 2ab + b^3, 3ab + 5b^3, 10b^2 + 5ab^4 + 7b^4), (28b^2 + 14ab^2 + 7, 4 + 2a + b^3, 6ab + 10b^3 + 16a + 15b^2 + 10b^3a + 5)\} \in S$.

We now find

A + B

$= \{(4 + 2a + b^3, 3a + 5b^2, 10b^3 + 5a + 7)\} + \{(2a + 1, b, b^4), (7b^2 + 1, 2b + 3, a)\}$

$= \{(5 + 4a + b^3, 3a + 5b^2 + b, 10b^3 + b^4 + 5a + 7), (2a + 7b^2 + 2, 3b + 3, a + b^4)\} \in S$.

This is the way operations are performed on S.

Further we see S has subset units and subset zero divisors.

For take A = $\{(a, b^2, a)\}$ and B = $\{(a, b^3, a)\} \in$ S.

We find A × B
$= \{(a, b^2, a)\} \times \{(a, b^3, a)\}$
$= \{(1, 1, 1)\} \in$ S is the subset unit.

Let
$A = \{(0, a, b), (0, 3a + 5b^2 + 8b, 9a + b^3), (0, 5b^4 + 13a + 9, 0)\}$
and

$B = \{(8a + b^3 + 1, 0, 0), (10a + 15ab + b^3, 0, 0), (12a + 3b^3 + 25ab^2, 0, 0)\} \in$ S.

We see $A \times B = \{(0, 0, 0)\}$ thus S has subset zero divisors.

S has subset subsemirings as well as subset semiring ideals.

We just give them for this S. Let P = {Collection of all subsets from the subset row matrix semiring $M_1 = \{(a, 0, 0) \mid a \in (Z^+ \cup \{0\}, D_{2,5}\} \subseteq M\} \subseteq S$, P is a subset row matrix subsemiring as well as subset semiring ideal of S.

Infact S has infinite number of subset row matrix subsemirings as well as subset row matrix semiring ideals. For take L = {Collection of all subsets from the row matrix subsemiring $V = \{(x, y, z) \mid x, y, z \in (Z^+ \cup \{0\})[G]$ where $G = \{b \mid b^5 = 1\} \subseteq D_{2.5}$ is a subgroup of $D_{2,5}\}\} \subseteq S$, L is a subset row matrix subsemiring of S.

Clearly S is not a subset row matrix semiring ideal of S. Thus we see S has infinite number of subset subsemirings which are not subset semiring ideals of S.

Example 3.34: Let S = {Collection of all subsets from the interval column matrix semiring;

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ [a_3, b_3] \\ [a_4, b_4] \end{bmatrix} \right| a_i, b_i \in \{Z^+ \cup \{0\}\}, 1 \leq i \leq 4\} \right\}$$

be the subset interval column matrix semiring of infinite order.

We see M has infinite number of subset zero divisors. M has also subset idempotents which are finite in number and has infinite number subset interval matrix subsemirings as well as infinite number of subset interval semiring ideals.

Example 3.35: Let S = {Collection of all subsets from the interval column matrices

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_9, b_9] \end{bmatrix} \right| a_i, b_i \in \langle Q^+ \cup I \cup \{0\}\rangle, 1 \leq i \leq 9 \right\}$$

be the subset interval column matrix semiring of infinite order. S has subset interval column matrix zero divisors, subset interval column matrix idempotents, subset interval column matrix subsemirings and subset interval column matrix semiring ideals.

Example 3.36: Let S = {Collection of all subsets from the interval column matrix semiring

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{12}, b_{12}] \end{bmatrix} \right| a_i, b_i \in (R^+ \cup \{0\})\,(g_1, g_2),\ g_1^2 = 0\ g_2^2 = g_2, \right.$$

$$\left. g_2g_1 = g_1g_2 = 0, 1 \leq i \leq 12\} \right\}$$

be the subset interval column matrix semiring of infinite order.

Let A = $\{ag_1 \mid a \in 3Z^+ \cup \{0\}\}$ and B = $\{bg_1 \mid b \in 5Z^+ \cup \{0\}\}$

$$\text{Suppose } S_A = \left\{ \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{12}, b_{12}] \end{bmatrix} \right| a_i, b_i \in A, 1 \leq i \leq 12 \right\} \text{ and}$$

$$S_B = \left\{ \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{12}, b_{12}] \end{bmatrix} \right| a_i, b_i \in B, 1 \leq i \leq 12 \right\} \text{ be subsets of S.}$$

Clearly S_A is a subset column interval matrix subsemiring and S_B is also a subset column interval matrix subsemiring.

Both S_A and S_B are not subset interval matrix semiring ideals of S.

Further for every $A_1 \in S_A$; $A_1 \times A_1 = \left\{ \begin{bmatrix} [0,0] \\ [0,0] \\ \vdots \\ [0,0] \end{bmatrix} \right\}$ and for

every $B_1 \in S_B$ we have $B_1 \times B_1 = \left\{ \begin{bmatrix} [0,0] \\ [0,0] \\ \vdots \\ [0,0] \end{bmatrix} \right\}$.

Also for every $A_1 \in S_A$ and $B_1 \in S_B$ we have

$$A_1 \times B_1 = \left\{\begin{bmatrix} [0,0] \\ [0,0] \\ \vdots \\ [0,0] \end{bmatrix}\right\}.$$

$$\text{Thus we have } S_A \times S_A = \left\{\begin{bmatrix} [0,0] \\ [0,0] \\ \vdots \\ [0,0] \end{bmatrix}\right\},$$

$$S_B \times S_B = \left\{\begin{bmatrix} [0,0] \\ [0,0] \\ \vdots \\ [0,0] \end{bmatrix}\right\} \text{ and } S_A \times S_B = \left\{\begin{bmatrix} [0,0] \\ [0,0] \\ \vdots \\ [0,0] \end{bmatrix}\right\}.$$

Thus S has infinite number of interval matrix subset zero divisors.

Take M = {Collection of all subsets from the interval column matrix subsemiring

$$P = \left\{\begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{12}, b_{12}] \end{bmatrix} \middle| \; a_i, b_i \in Q^+g_1 \cup \{0\}, 1 \leq i \leq 12\}\right\} \subseteq S$$

be a subset interval column matrix subsemiring.

P is also a subset interval column matrix semiring ideal of S.

Consider N = {Collection of all subsets from the interval column matrix subsemiring

$$T = \left\{ \left[\begin{matrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{12}, b_{12}] \end{matrix}\right] \middle| \; a_i, b_i \in Q^+g_2 \cup \{0\}, 1 \leq i \leq 12 \right\} \subseteq S$$

be the subset interval column matrix subsemiring.

N is also a subset interval column matrix semiring ideal of S.

Example 3.37: Let S = {Collection of all subsets from the interval column matrix semiring

$$M = \left\{ \left[\begin{matrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_9, b_9] \end{matrix}\right] \middle| \; a_i, b_i \in (\langle Z^+ \cup I \cup \{0\}\rangle)\,(g_1, g_2),\; g_1^2 = 0, \right.$$

$$\left. g_2^2 = g_2,\, g_1g_2 = g_2g_1 = 0,\, 1 \leq i \leq 9 \right\}\}$$

be the subset interval column matrix semiring of infinite order.

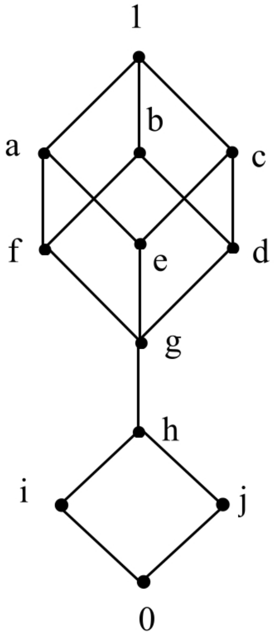
S has infinite number of subset interval column matrix subsemirings, also S has infinite number of subset interval column matrix zero divisors but has only finite number of subset idempotents.

All these subset column interval matrix semirings are commutative and of course the product is the natural product $\times_n$.

We now proceed onto give examples of both finite subset interval column matrix semirings and infinite subset interval column matrix semirings which are non commutative.

Example 3.38: Let S = {collection of all subsets from the interval column matrix semiring

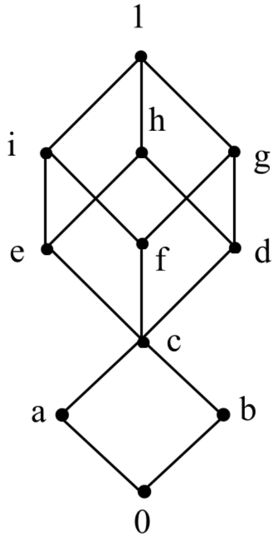
$$M = \left\{ \left[\begin{array}{c} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_9, b_9] \end{array} \right] \right| a_i, b_i \in L =$$



$1 \leq i \leq 9\}\}$ be the subset interval column matrix semiring of finite order. S has subset idempotents and subset zero divisors.

Example 3.39: Let S = {Collection of all subsets from the interval semiring

$$M = \left\{ \left[\begin{array}{c} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{12}, b_{12}] \end{array} \right] \right| a_i, b_i \in L =$$



$1 \leq i \leq 12\}\}$ be the subset interval column matrix semiring of finite order.

Example 3.40: Let S = {Collection of all subsets from the interval column matrix semiring

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{10}, b_{10}] \end{bmatrix} \right| a_i, b_i \in (Z^+ \cup \{0\})S_3, 1 \leq i \leq 10 \right\}\}$$

be the subset interval matrix semiring of infinite order which is non commutative under natural product $\times_n$.

Example 3.41: Let S = {Collection of all subsets from the interval matrix column semiring

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ [a_3, b_3] \\ [a_4, b_4] \end{bmatrix} \right| a_i, b_i \in (Q^+ \cup \{0\})D_{27}; 1 \leq i \leq 4 \right\}\}$$

be the subset interval column matrix semiring; which is non commutative of infinite order under natural product $\times_n$.

$$\text{Let } A = \left\{ \begin{bmatrix} [3a+2, 5b] \\ [6b+8, 0] \\ [a+2b, 5b^2] \\ [3a+2ab^2, ab] \end{bmatrix} \right\} \text{ and}$$

$$B = \left\{ \begin{bmatrix} [0, 3a+2b] \\ [5b, 6a+5b+3ab] \\ [2ab+b^2, 3a+5b+3ab] \\ [0, ab+3ab^2] \end{bmatrix} \right\} \in S.$$

We find

$$A + B = \left\{ \begin{bmatrix} [3a+2, 5b] \\ [6b+8, 0] \\ [a+2b, 5b^2] \\ [3a+2ab^2, ab] \end{bmatrix} \right\} + \left\{ \begin{bmatrix} [0, 3a+2b] \\ [5b, 6a+5b+3ab] \\ [2ab+b^2, 3a+5b+3ab] \\ [0, ab+3ab^2] \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} [3a+2, 7b+3a] \\ [11b+8, 6a+5b+3ab] \\ [a+2b+2ab+b^2, 5b^2+3a+5b+3ab] \\ [3a+2ab^2, 2ab+3ab^2] \end{bmatrix} \right\} \in S.$$

Now we find the product

$$A \times B = \left\{ \begin{bmatrix} [3a+2, 5b] \\ [6b+8, 0] \\ [a+2b, 5b^2] \\ [3a+2ab^2, ab] \end{bmatrix} \right\} \times \left\{ \begin{bmatrix} [0, 3a+2b] \\ [5b, 6a+5b+3ab] \\ [2ab+b^2, 3a+5b+3ab] \\ [0, ab+3ab^2] \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} [3a+2, 5b] \times [0, 3a+2b] \\ [6b+8, 0] \times [5b, 6a+5b+3ab] \\ [a+2b, 5b^2] \times [2ab+b^2, 3a+5b+3ab] \\ [3a+2ab^2, ab] \times [0, ab+3ab^2] \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} [0, 15ba + 10b^2] \\ [30b^2 + 40b, 0] \\ [2b + 4a + ab^2 + 2b^3, 15b^2a + 25b^3 + 15ba] \\ [0, 1 + b] \end{bmatrix} \right\} \in S.$$

This is the way operation + and × are performed on S.

Example 3.42: Let S = {Collection of all subsets from the interval column matrix semiring

$$M = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_8, b_8] \end{bmatrix} \middle| \; a_i, b_i \in (R^+ \cup \{0\})S_4, 1 \leq i \leq 8 \right\}\}$$

be the subset interval column matrix semiring.

S is a non commutative infinite subset interval column matrix semiring.

S has infinite number of subset zero divisors. S has subset idempotents.

$$\text{Let } A = \left\{ \begin{bmatrix} [0,0] \\ [1,0] \\ [0,1] \\ [1,1] \\ [0,0] \\ [1,0] \\ [1,1] \\ [0,0] \end{bmatrix} \right\} \in S.$$

A × A = A is a subset idempotent of S.

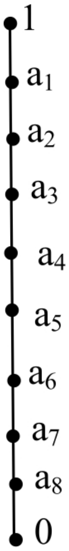
$$\text{Let } B = \left\{\begin{bmatrix} [1,1] \\ [0,0] \\ [0,0] \\ [0,0] \\ [0,0] \\ [0,0] \\ [0,0] \\ [1,1] \end{bmatrix}\right\} \in S.$$

B × B = B is again a subset idempotent in S. S has only a finite number of subset idempotents.

Example 3.43: Let S = {Collection of all subsets from the

$$\text{interval column matrix semiring } M = \left\{\begin{bmatrix} [a_1,b_1] \\ [a_2,b_2] \\ \vdots \\ [a_5,b_5] \end{bmatrix} \middle|\ a_i, b_i \in LA_4\right.$$

where L =



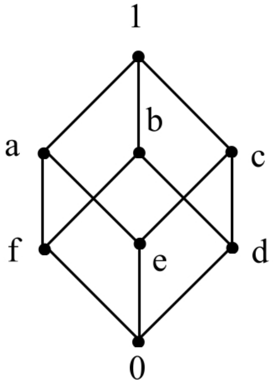
$1 \leq i \leq 5\}$ be the subset interval column matrix semiring which is non commutative but of finite order.

Clearly S has no subset nilpotents of order two but S has subset zero divisors.

Example 3.44: Let S = {Collection of all subsets from the

interval column matrix semiring $M = \left\{ \left. \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_9, b_9] \end{bmatrix} \right| a_i, b_i \in LD_{2.6}; \right.$

L =



$1 \leq i \leq 9\}$ be the subset interval column matrix semiring of finite order which is non commutative.

We can have several such subset interval column matrix semirings which are commutative and non commutative.

Now we just proceed onto describe subset interval m × n matrix semiring.

Example 3.45: Let S = {Collection of all subsets from the interval 3 × 5 matrix semiring

$$S = \left\{ \left. \begin{bmatrix} [a_1, b_1] & \ldots & [a_5, b_5] \\ [a_6, b_6] & \ldots & [a_{10}, b_{10}] \\ [a_{11}, b_{11}] & \ldots & [a_{15}, b_{15}] \end{bmatrix} \right| a_i, b_i \in Q^+ \cup \{0\}, 1 \leq i \leq 15 \right\}\}$$

be the subset interval 3 × 5 matrix semiring of infinite order under natural product $\times_n$ of matrices.

Clearly S is commutative.

Example 3.46: Let S = {Collection of all subsets from the interval 5 × 2 matrix semiring

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \\ \vdots & \vdots \\ [a_9, b_9] & [a_{10}, b_{10}] \end{bmatrix} \middle| \; a_i, b_i \in Z^+ \cup \{0\}, 1 \le i \le 10 \right\}\}$$

be the subset interval 5 × 2 matrix semiring of infinite order which is commutative and has infinite number of subset zero divisors but has only finite number of subset idempotents.

Example 3.47: Let S = {Collection of all subsets from the interval 5 × 5 matrix semiring

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & \dots & [a_5, b_5] \\ [a_6, b_6] & \dots & [a_{10}, b_{10}] \\ \vdots & \dots & \vdots \\ [a_{21}, b_{21}] & \dots & [a_{25}, b_{25}] \end{bmatrix} \middle| \; a_i, b_i \in \langle Z^+ \cup I \rangle \cup \{0\}, \right.$$

$$\left. 1 \le i \le 25 \right\}\}$$

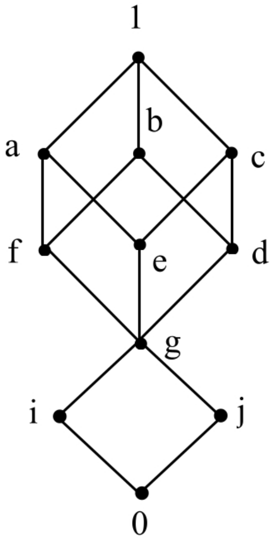
be the subset interval square matrix semiring of infinite order.

S has infinite number subset interval matrix zero divisors but has only finite number of subset interval matrix idempotents.

S has also subset interval matrix subsemirings and subset interval matrix semiring ideals.

Example 3.48: Let S = {Collection of all subsets from the 3×10 interval matrix semiring

$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] & [a_2, b_2] & \ldots & [a_{10}, b_{10}] \\ [a_{11}, b_{11}] & [a_{12}, b_{12}] & \ldots & [a_{20}, b_{20}] \\ [a_{21}, b_{21}] & [a_{22}, b_{22}] & \ldots & [a_{30}, b_{30}] \end{bmatrix} \right| a_i, b_i \in L = \right.$$



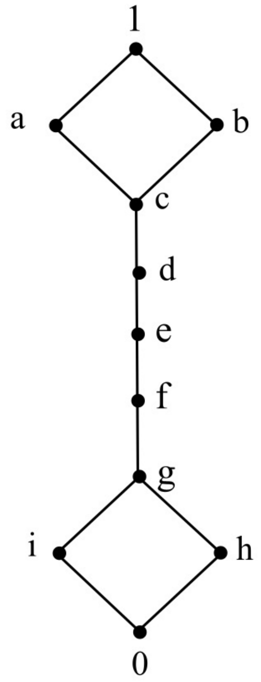
$1 \leq i \leq 30\}\}$ be the subset interval matrix semiring.

Clearly S is of finite order and S is commutative.

Example 3.49: Let S = {Collection of all subsets from the

$$\text{interval matrix semiring } M = \left\{ \left. \begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ \vdots & \vdots \\ [a_{11}, b_{11}] & [a_{12}, b_{12}] \end{bmatrix} \right| a_i, b_i \in \right.$$

L =



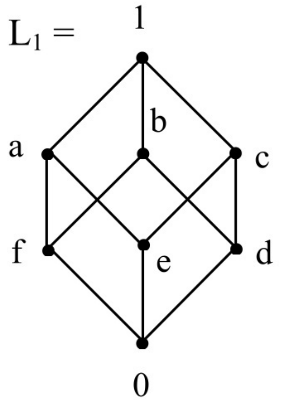
$1 \leq i \leq 12\}\}$ be subset interval matrix semiring of finite order. S has several subset idempotents only a few subset zero divisors.

Example 3.50: Let S = {Collection of all subsets from the interval matrix semiring

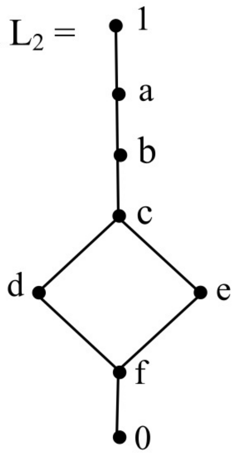
$$M = \left\{ \left. \begin{bmatrix} [a_1, b_1] & [a_2, b_2] & \ldots & [a_{12}, b_{12}] \\ [a_{13}, b_{13}] & [a_{14}, b_{14}] & \ldots & [a_{24}, b_{24}] \\ [a_{25}, b_{25}] & [a_{26}, b_{26}] & \ldots & [a_{36}, b_{36}] \end{bmatrix} \right| a_i, b_i \in L = L_1 \times L_2 \right.$$

where

L_1 =



and L_2 =



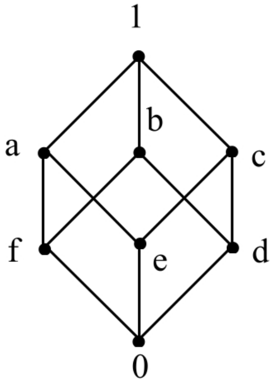
$1 \leq i \leq 36\}\}$ be the subset interval matrix semiring of finite order.

S is commutative has subset zero divisors and subset idempotents.

Example 3.51: Let S = {Collection of all subsets from the subset interval matrix semiring

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & \ldots & [a_{13}, b_{13}] \\ [a_{14}, b_{14}] & \ldots & [a_{26}, b_{26}] \end{bmatrix} \middle| \; a_i, b_i \in L \times Z^+ \cup \{0\} \text{ where} \right.$$

L =



$1 \leq i \leq 26\}$ be a subset interval matrix semiring of infinite order.

Now we proceed onto describe a few non commutative subset interval matrix semiring.

Example 3.52: Let S = {Collection of all subsets from the interval matrix semiring

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \\ [a_5, b_5] & [a_6, b_6] \end{bmatrix} \middle| \; a_i, b_i \in (Z^+ \cup \{0\})S_3, 1 \leq i \leq 6\}\} \right.$$

be the subset interval matrix semiring of infinite order under the natural product $\times_n$. S has subset zero divisors and subset units.

Clearly $\left\{\begin{bmatrix} [0,0] & [0,0] \\ [0,0] & [0,0] \\ [0,0] & [0,0] \end{bmatrix}\right\}$ is the subset interval zero matrix.

$\left\{\begin{bmatrix} [1,1] & [1,1] \\ [1,1] & [1,1] \\ [1,1] & [1,1] \end{bmatrix}\right\}$ is the subset interval unit matrix of S.

Let A = $\left\{\begin{bmatrix} [p_1,1] & [1,p_1] \\ [p_2,1] & [p_1,p_2] \\ [p_3,p_3] & [1,p_3] \end{bmatrix}\right\}$ $\in$ S is such that

$$A \times A = \left\{\begin{bmatrix} [p_1,1] & [1,p_1] \\ [p_2,1] & [p_1,p_2] \\ [p_3,p_3] & [1,p_3] \end{bmatrix}\right\} \times \left\{\begin{bmatrix} [p_1,1] & [1,p_1] \\ [p_2,1] & [p_1,p_2] \\ [p_3,p_3] & [1,p_3] \end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix} [p_1,1]\times[p_1,1] & [1,p_1]\times[1,p_1] \\ [p_2,1]\times[p_2,1] & [p_1,p_2]\times[p_1,p_2] \\ [p_3,p_3]\times[p_3,p_3] & [1,p_3]\times[1,p_3] \end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix} [1,1] & [1,1] \\ [1,1] & [1,1] \\ [1,1] & [1,1] \end{bmatrix}\right\}$$

(since $p_1^2 = 1$, $p_2^2 = 1$, $p_3^2 = 1$ where $p_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$,

$p_2 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$ and $p_3 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$).

Thus A is a subset interval matrix unit of S.

$$A = \left\{\begin{bmatrix} [0,1] & [0,3p_1+p_2+p_3] \\ [p_4+p_5,0] & [5p_4+8p_5,0] \\ [0,0] & [8p_5+9p_1,0] \end{bmatrix}\right\}$$

and

$$B = \left\{\begin{bmatrix} [8p_1+9p_3+p_2,0] & [21p_4+81p_5,0] \\ [0,10p_1+27p_5] & [0,40p_3+59p_4+p_5] \\ [98p_1+100p_2+43p_5,28p_1] & [0,40p_2+38p_4+p_3] \end{bmatrix}\right\} \in S$$

is such that A × B = B × A

$$= \left\{\begin{bmatrix} [0,0] & [0,0] \\ [0,0] & [0,0] \\ [0,0] & [0,0] \end{bmatrix}\right\} \text{ is the subset zero divisor of S.}$$

(Here $p_4 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ and $p_5 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \in S_3$).

Clearly S is non commutative and of infinite order.
Take

$$A = \left\{\begin{bmatrix} [p_1,1] & [p_2,0] \\ [0,p_3] & [1,1] \\ [0,p_1] & [p_2,p_3] \end{bmatrix}\right\} \text{ and } B = \left\{\begin{bmatrix} [p_2,1] & [p_3,1] \\ [0,p_4] & [0,p_2] \\ [1,p_2] & [p_4,p_1] \end{bmatrix}\right\} \in S.$$

We now find

$$A \times B = \left\{\begin{bmatrix} [p_1,1] & [p_2,0] \\ [0,p_3] & [1,1] \\ [0,p_1] & [p_2,p_3] \end{bmatrix}\right\} \times \left\{\begin{bmatrix} [p_2,1] & [p_3,1] \\ [0,p_4] & [0,p_2] \\ [1,p_2] & [p_4,p_1] \end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix} [p_1,1]\times[p_2,1] & [p_2,0]\times[p_3,1] \\ [0,p_3]\times[0,p_4] & [1,1]\times[0,p_2] \\ [0,p_1]\times[1,p_2] & [p_2,p_3]\times[p_4,p_1] \end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix} [p_5,1] & [p_5,0] \\ [0,p_2] & [0,p_2] \\ [1,p_5] & [p_1,p_5] \end{bmatrix}\right\} \quad \ldots \text{ I}$$

Consider

$$B \times A = \left\{\begin{bmatrix} [p_2,1] & [p_3,1] \\ [0,p_4] & [0,p_2] \\ [1,p_2] & [p_4,p_1] \end{bmatrix}\right\} \times \left\{\begin{bmatrix} [p_1,1] & [p_2,0] \\ [0,p_3] & [1,1] \\ [0,p_1] & [p_2,p_3] \end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix} [p_2,1]\times[p_1,1] & [p_3,1]\times[p_2,0] \\ [0,p_4]\times[0,p_3] & [0,p_2]\times[1,1] \\ [1,p_2]\times[0,p_1] & [p_4,p_1]\times[p_2,p_3] \end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix} [p_4,1] & [p_4,0] \\ [0,p_1] & [0,p_2] \\ [1,p_4] & [p_3,p_4] \end{bmatrix}\right\} \quad \ldots \text{ II}$$

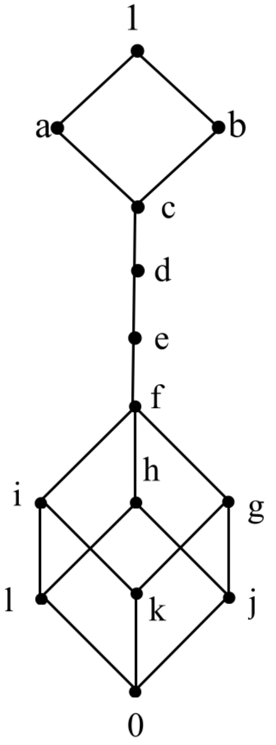
Clearly I and II are distinct so A × B ≠ B × A in general for A, B ∈ S.

Thus S is non commutative subset interval matrix semiring under the natural product $\times_n$ of interval matrices.

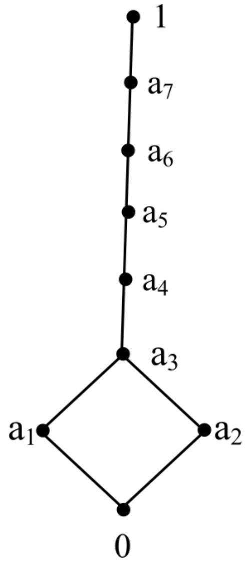
Example 3.53: Let S = {Collection of all subsets from the interval matrix semiring

$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \\ \vdots & \vdots \\ [a_9, b_9] & [a_{10}, b_{10}] \end{bmatrix} \right| a_i, b_i \in (L_1 \times L_2)S_3; \text{ where}$$

$L_1 =$



and $L_2 =$



$1 \leq i \leq 10\}\}$ be the subset interval matrix semiring of finite order which is non commutative.

S has subset zero divisors and subset idempotents.

S has also subset interval matrix subsemirings and subset interval matrix semiring ideals.

Example 3.54: Let S = {Collection of all subsets from the interval matrix semiring

$$M = \left\{ \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ [a_3, b_3] \\ [a_4, b_4] \end{bmatrix} \middle| \; a_i, b_i \in (Z^+ \cup \{0\}), (S_3 \times A_4 \times D_{2,7}), \right.$$

$$\left. 1 \leq i \leq 4 \right\}\}$$

be the subset interval matrix non commutative semiring of infinite order.

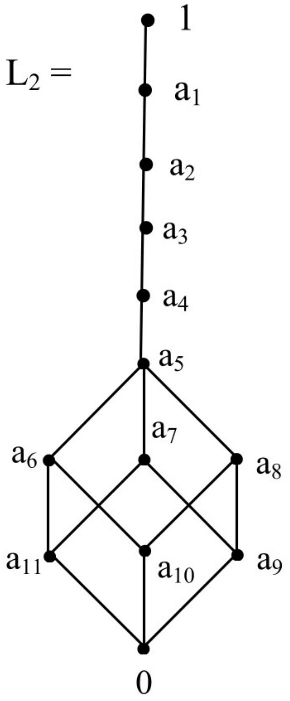
S has subset units, subset zero divisors and subset idempotents.

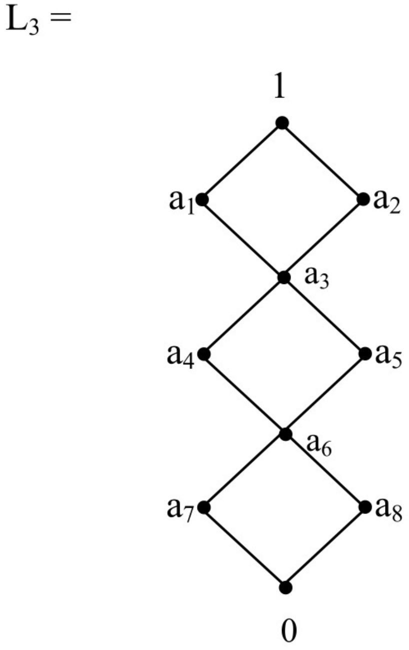
Infact S has infinite number of subset interval matrix subsemirings and subset interval matrix semiring ideals.

Example 3.55: Let S = {Collection of all subsets from the interval matrix semiring

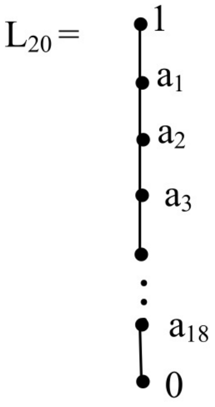
$$M = \left\{ \begin{bmatrix} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \\ [a_5, b_5] & [a_6, b_6] \\ [a_7, b_7] & [a_8, b_8] \\ [a_9, b_9] & [a_{10}, b_{10}] \end{bmatrix} \middle| \; a_i, b_i \in (L_1 \times L_2 \times L_3 \times L_4) A_4 \right.$$

where L_1 is a Boolean algebra of order 16,





and L_4 is a chain lattice.



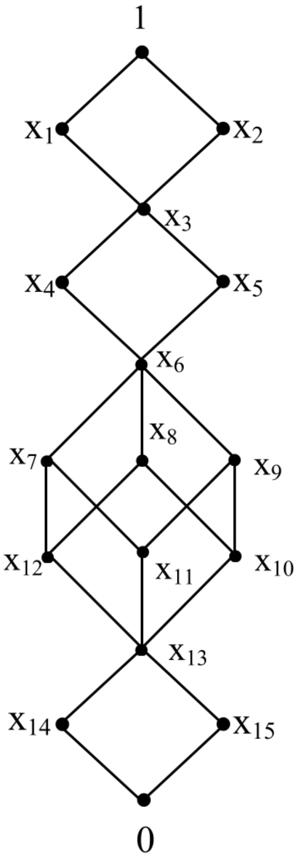
$1 \leq i \leq 10\}\}$ be the subset interval matrix non commutative semiring of finite order.

This has lots of subset zero divisors, subset units and subset idempotents.

Example 3.56: Let S = {Collection of all subsets from the interval matrix semiring

$$M = \left\{ \left[\begin{array}{cc} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \\ [a_5, b_5] & [a_6, b_6] \end{array} \right] \right| a_i, b_i \in LS_4 \text{ where}$$

L =



$1 \leq i \leq 6\}$ be the subset interval matrix semiring of finite order. S is non commutative and has subset idempotents and subset zero divisors. S has also subset semiring ideals.

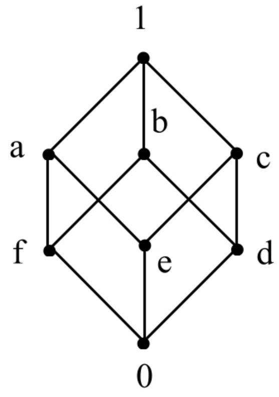
Example 3.57: Let S = {Collection of all subsets from the interval matrix semiring

$$M = \left\{ \left[\begin{array}{cc} [a_1, b_1] & [a_2, b_2] \\ [a_3, b_3] & [a_4, b_4] \\ \vdots & \vdots \\ [a_{19}, b_{19}] & [a_{20}, b_{20}] \end{array} \right] \right| a_i, b_i \in (L_1 \times L_2)\,(S_3 \times D_{2,7});$$

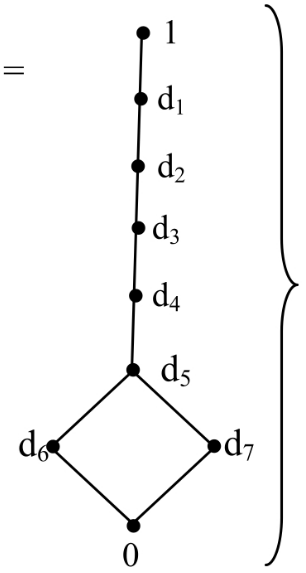
$$1 \leq i \leq 20$$

where

$L_1 =$



and $L_2 =$



be the subset interval matrix semiring of finite order which is non commutative.

Next we proceed onto describe subset polynomial semirings of type II by examples.

Example 3.58: Let S = {Collection of all subsets from the polynomial semiring

$$M = \left\{ \sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in Z^+ \cup \{0\} \right\}\}$$

be the subset polynomial semiring of type II.

Such study is made in [26].

We see this is an infinite commutative semiring which has subset semiring ideals and subset subsemirings.

However we see this subset semiring has no subset zero divisors and is a strict semiring which is a subset semifield.

It is also important to observe if S = {Collection of all subsets from the semiring $Z^+ \cup \{0\}$ or $Q^+ \cup \{0\}$ or $R^+ \cup \{0\}$} then S is a subset semifield.

For if A, B ∈ S we see $A + B \neq \{0\}$ if $A \neq \{0\}$ and $B \neq \{0\}$ further $A \times B \neq \{0\}$ if $A \neq \{0\}$ and $B \neq \{0\}$.

Thus we can have subset semifields. Infact we have a class of such subset semifields.

Example 3.59: Let
S = {Collection of all subsets from $(Q^+ \cup \{0\})[x]$} be the subset polynomial semiring of type II. S has no subset zero divisors. Further S is a subset semifield.

Suppose
$A=\{3x^2 + 2, 10x^3 + 4x + 1\}$ and $B = \{x^7 + 3x + 1, 3x^8 + 4\} \in S$.

We now show A + B = B + A and A × B = B × A.

A + B

$$= \{3x^2 + 2, 10x^3 + 4x + 1\} + \{x^7 + 3x + 1, 3x^8 + 4\}$$

$$= \{3x^2 + 2 + x^7 + 3x + 1, 10x^3 + 4x + 1 + x^7 + 3x + 1, 10x^3 + 4x + 1 + 3x^8 + 4, 3x^2 + 2 + 3x^8 + 4\}$$

$$= \{x^7 + 3x^2 + 3x + 3, x^7 + 10x^3 + 7x + 2, 3x^8 + 10x^3 + 4x + 5, 3x^8 + 3x^2 + 6\} \in S.$$

Easily verified A + B = B + A, under addition infact for all A, B ∈ S.

Consider
A × B

$$= \{3x^2 + 2, 10x^3 + 4x + 1\} \times \{x^7 + 3x + 1, 3x^8 + 4\}$$

$$= \{3x^2 + 2 \times x^7 + 3x + 1, 3x^2 + 2 \times 3x^8 + 4, 10x^3 + 4x + 1 \times x^7 + 3x + 1, 10x^3 + 4x + 1 \times 3x^8 + 4\}$$

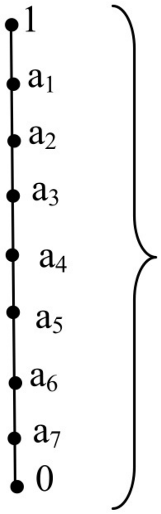
$$= \{21x^9 + 9x^3 + 3x^2 + 2x^7 + 6x + 2,\ 9x^{10} + 6x^8 + 12x^2 + 8,\ 10x^{10} + 4x^8 + x^7 + 30x^4 + 12x^2 + 3x + 10x^3 + 4x + 1,\ 30x^{11} + 12x^9 + 3x^8 + 40x^3 + 16x + 4\}$$

$$= \{21x^9 + 2x^7 + 9x^3 + 3x^2 + 6x + 2,\ 9x^{10} + 6x^8 + 12x^2 + 8,\ 10x^{10} + 4x^8 + x^7 + 3x^4 + 10x^3 + 12x^2 + 7x + 1,\ 30x^{11} + 12x^9 + 3x^8 + 40x^3 + 16x + 4\} \in S.$$

It is easily verified $A \times B = B \times A$ for all $A, B \in S$ as basically polynomial multiplication is commutative.

By using the notion of subset polynomial semiring of type II we get subset semifields of infinite order.

Example 3.60: Let S = {Collection of all subsets from the polynomial semiring L[x] where L =



be the subset polynomial semiring.

S is again a subset semifield.

We have the following theorem.

THEOREM 3.3: *Let S = {Collection of all subsets from the polynomial ring ($R^+ \cup \{0\}$)[x] (or ($Q^+ \cup \{0\}$)[x], ($Z^+ \cup \{0\}$)[x] or C_n[x]; $n < \infty$, C_n a chain lattice of length n)} be the subset polynomial semiring of type II.*

S is a subset polynomial semifield.

The proof is direct and hence left as an exercise to the reader.

It is pertinent to keep on record that we do not have subset field of polynomials further we do not have subset semifields if we build subset polynomials from the distributive lattices which are not are chain lattices and also over semirings ($Z^+ \cup \{0\}$) (g) [x] where $g^2 = 0$ and so on.

We will just illustrate this situation by an example or two.

Example 3.61: Let S = {Collection of all subsets from the polynomial semiring ($Z^+ \cup \{0\}$)(g_1,g_2)[x]; where $g_1^2 = 0$, $g_2^2 = g_2$ and $g_1g_2 = g_2g_1 = 0$}} be the subset polynomial semiring.

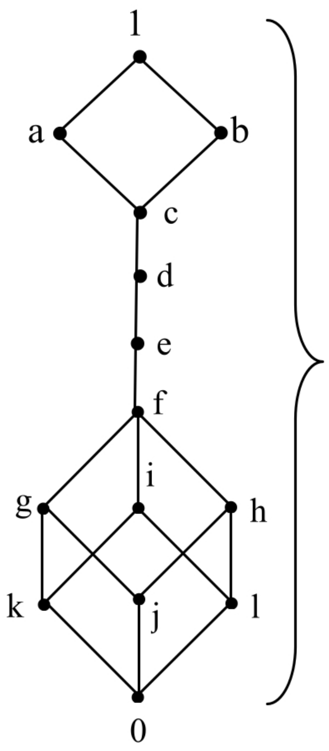
Clearly S is not a subset semifield, for if A = {$8g_1$} and B = {$18g_1x^8 + 10x^3g_1 + 12g_1$} ∈ S, we see A × B = {0} so S is not a subset semifield only a subset semiring.

Example 3.62: Let S = {Collection of all subsets from the polynomial semiring

$$M = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in L[x] \right.$$

where L is a distributive lattice given in the following:

L =



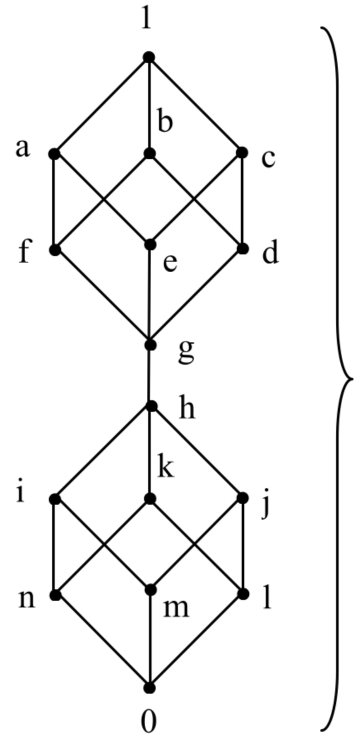
be the subset polynomial semiring. Clearly S is not a subset semifield.

Take A = $\{kx^{10} + jx^5 + k, jx^{18} + kx^{12} + j\}$ and B = $\{lx^{20} + lx^{14}, l^{28}\} \in$ S. We see A × B = {0}.

Thus S is only a subset semiring and is not a subset semifield.

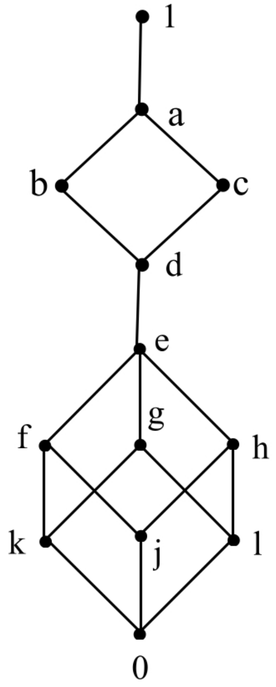
Example 3.63: Let S = {Collection of all subsets from the polynomial semiring

$$M = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in L[x] \text{ where } L = \right.$$



be the subset polynomial semiring. S is not a subset polynomial semifield as S has subset zero divisors. We see if L has zero divisors then S also has subset zero divisors.

Example 3.64: Let S = {Collection of all subsets from the polynomials semiring $(L_1 \times L_2)$ [x]; where L_1 =



and L = Boolean algebra of order 2^5}} be the subset polynomial semiring. S is not a subset semifield.

Example 3.65: Let S = {Collection of all subsets from the polynomial semiring

$$M = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in (Q^+ \cup \{0\})\,(g_1, g_2, g_3),\ g_1^2 = 0,\ g_2^2 = g_2,\right.$$

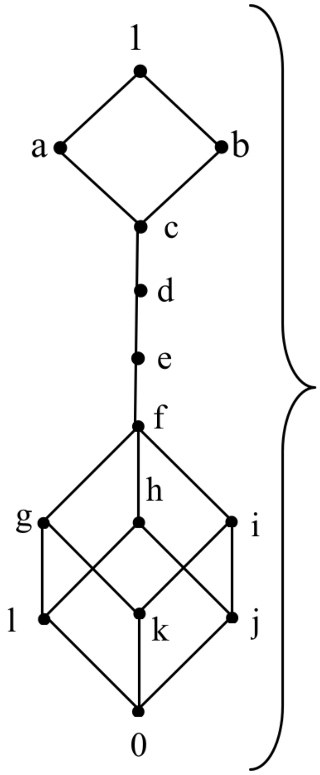
$$\left. g_3^2 = 0,\ g_ig_j = g_jg_i = 0,\ 1 \leq i, j \leq 3 \right\}\}$$

be the subset polynomial semiring which is not a subset semifield.

Example 3.66: Let S = {Collection of all subsets from the polynomial semiring

$$M = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in L_1 \times L_2 \times L_3 \text{ where} \right.$$

L_1 = a Boolean algebra of order 2^4, $L_2 = C_6$ a chain lattice and L_3 =



be the subset polynomial semiring which is not a subset polynomial semifield.

Now we proceed onto describe subset interval polynomial semirings.

Example 3.67: Let S = {Collection of all subsets from the interval polynomial semiring

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in R^+ \cup \{0\} \right\}\}$$

be the subset interval polynomial semiring which is not a semifield.

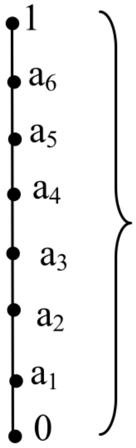
Example 3.68: Let S = {Collection of all subsets from the interval polynomial semiring

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in Q^+ \cup \{0\} \right\}\}$$

be the subset interval polynomial semiring of M. S is not a semifield.

Example 3.69: Let S = {Collection of all subsets from the interval polynomial semiring

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in L = \right.$$



be the subset interval polynomial semiring.

L is a chain lattice and has no zero divisors but S has subset zero divisors.

Take $A = \{[0, a_1]x^8 + [0, a_3]x^5 + [0, a_2]x^3 + [0, a_1], [0, a_6]x^7 + [0, a_2]\}$ and
$B = \{[a_1, 0]x^9 + [a_3, 0], [a_3, 0]x^5 + [a_1, 0]x^2 + [a_5, 0]\} \in S$.

We see $A \times B = \{[0, 0]\}$.

Thus S has subset zero divisors so S is not a subset interval polynomial semifield only a subset interval polynomial semiring.

Example 3.70: Let S = {Collection of all subsets from the polynomial semiring

$$M = \left\{\sum_{i=0}^{\infty}[a_i, b_i]x^i \,\middle|\, a_i, b_i \in Q^+ \cup \{0\}\right\}\}$$

be the subset interval polynomial semiring. S is not a subset interval polynomial semifield. Consider A_1 = {Collection of all subsets from the interval polynomial subsemiring

$$P_1 = \left\{\sum_{i=0}^{\infty}[0, a_i]x^i \,\middle|\, a_i \in Q^+ \cup \{0\}\right\} \subseteq M\} \subseteq S$$

is a subset interval polynomial subsemiring of S.

A_2 = {Collection of all subsets from the interval polynomial subsemiring

$$P_2 = \left\{\sum_{i=0}^{\infty}[a_i, 0]x^i \,\middle|\, a_i \in Q^+ \cup \{0\}\right\} \subseteq M\} \subseteq S$$

is a subset interval polynomial subsemiring of S.

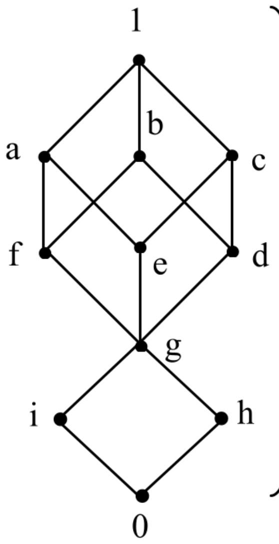
We see A_1 and A_2 are subset interval polynomial semiring ideals of S. Clearly $A_1 \times A_2 = \{[0, 0]\}$.

That is every element in A_1 annihilates every element in A_2 and vice versa.

If in the example 3.70; $Q^+ \cup \{0\}$ is replaced by any semiring or any semifield or by a chain lattice or any other distributive lattice still the results hold good.

Example 3.71: Let S = {Collection of all subsets from the interval polynomial semiring

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in L = \right.$$



$$\left. \vphantom{\sum_{i=0}^{\infty}} \right\}\Bigg\}$$

be the subset interval polynomial semiring. Clearly S is not a subset semifield.

Inview of this we have the following theorem.

THEOREM 3.4: *Let S = {Collection of all subsets from the interval polynomial semiring*

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in Z^+ \cup \{0\} \text{ (or } Q^+ \cup \{0\} \text{ or } R^+ \cup \{0\}\right.$$

or $(Q^+ \cup \{0\})(g)$ $(g^2 = 0)$, *L a distributive lattice and so on)}} be the subset interval polynomial semiring.*

(i) S has infinite number of subset interval polynomial zero divisors.

(ii) S has two subset interval polynomial semiring ideals A_1 and A_2 such that $A_1 \times A_2 = \{[0, 0]\}$.

The proof is direct and hence left as an exercise to the reader.

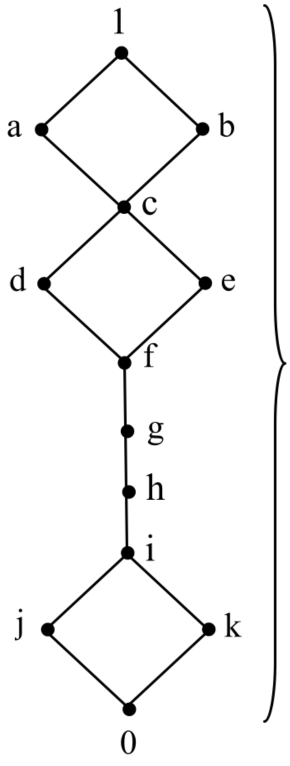
We have already given examples of these.

Now we proceed onto give examples of finite subset interval polynomial semirings of type II.

Example 3.72: Let S = {Collection of all subsets from the interval polynomial semiring

$$M = \left\{ \sum_{i=0}^{8} [a_i, b_i]x^i \,\middle|\, a_i, b_i \in L, 0 \le i \le 8 \text{ and } x^9 = 1 \right\}$$

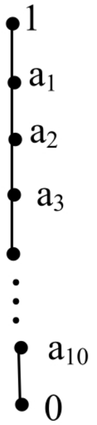
and L is the lattice which is as follows:



be the subset interval polynomial semiring of finite order. S is commutative but has subset zero divisors.

Example 3.73: Let S = {Collection of all subsets from the interval polynomial semiring

$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i]x^i \right| a_i, b_i \in L$ as follows:



$\}\}$

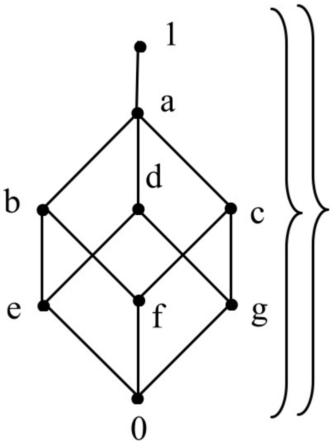
be the subset interval polynomial semiring of finite order which is commutative.

S has subset interval polynomial semiring ideals as well as subset zero divisors and subset idempotents.

Example 3.74: Let S = {Collection of all subsets from the interval polynomial semiring

$$M = \left\{ \sum_{i=0}^{3} [a_i, b_i]x^i \right| a_i, b_i \in LS_4;\ x^4 = 1;\ 0 \leq i \leq 3\}$$

and L is as follows:



be the subset interval polynomial semiring of type II of finite order.

The main feature about this S is that S is a non commutative subset polynomial interval semiring of finite order.

Example 3.75: Let S = {Collection of all subsets from the interval polynomial semiring

$$M = \left\{ \sum_{i=0}^{7} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in (L_1 \times L_2)\, A_4 \text{ where } L_1 \text{ is a} \right.$$

Boolean algebra of order 2^4 and L_2 is chain lattice C_{10}; $0 \leq i \leq 7$} and $x^8 = 1$}} be the finite interval subset polynomial semiring of finite order which is non commutative.

If $L_1 \times L_2$ is replaced by $Z^+ \cup \{0\}$ or $\langle Z^+ \cup I \rangle \cup [0]$ or $Q^+ \cup \{0\}$ and so on we get non commutative infinite subset interval polynomial semirings of type II.

We suggest the following problems for this chapter.

Problems

1. What are the special features enjoyed by subset semirings of type II?

2. Distinguish between the subset semirings of type I and subset semirings of type II.

3. Give some examples of subset semiring of finite order which is non commutative.

4. Let S = {Collection of all subsets from the semiring $S = Z^+ \cup \{0\}$} be the subset semiring of type II.

   (i) Does S contain subset zero divisors?
   (ii) Can S have subset idempotents?
   (iii) Can S have subset subsemirings which are not subset semiring ideals?
   (iv) Is S a S-subset semiring?
   (v) Can S have S-subset semiring ideals?

5. Let S_I = {Collection of all subsets from the semiring $\langle Z^+ \cup I\rangle \cup \{0\}$} be the neutrosophic subset semiring.

   Study questions (i) to (v) of problem 4 for this S_I.

6. Let S = {Collection of all subsets from the semiring $(Z^+ \cup \{0\})(g)$ where $g^2 = 0$} be the subset semiring of type II.

   Study questions (i) to (v) of problem 4 for this S.

7. Let S_1 = {Collection of all subsets from the semiring $(Z^+ \cup \{0\})$ (g) where $g^2 = g$} be the subset semiring.

   Study questions (i) to (v) of problem 4 for this S_1.

   Is S_I in problem 5 isomorphic with this S_1.

8. Let S = {Collection of all subsets from the subset semiring $(Z^+ \cup \{0\})(g_1, g_2)$ where $g_1^2 = 0$, $g_2^2 = g_2$, $g_1g_2 = g_2g_1 = 0$} be the subset semiring of type II.

   Study questions (i) to (v) of problem 4 for this S.

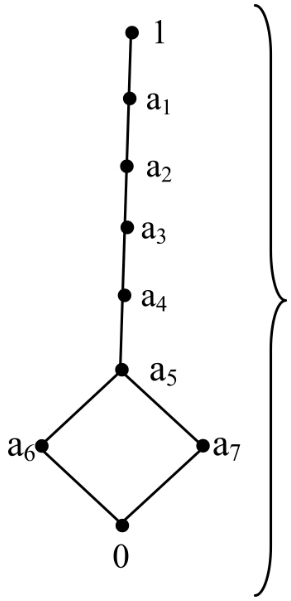
9. Let S = {Collection of all subsets of the semiring $R^+ \cup \{0\}$} be the subset semiring of type II.

   Study questions (i) to (v) of problem 4 for this S.

10. Let S = {Collection of all subsets from the semiring L = a Boolean algebra of order 2^8} be the subset semiring of type II of finite order.

    Study questions (i) to (v) of problem (4) for this S.

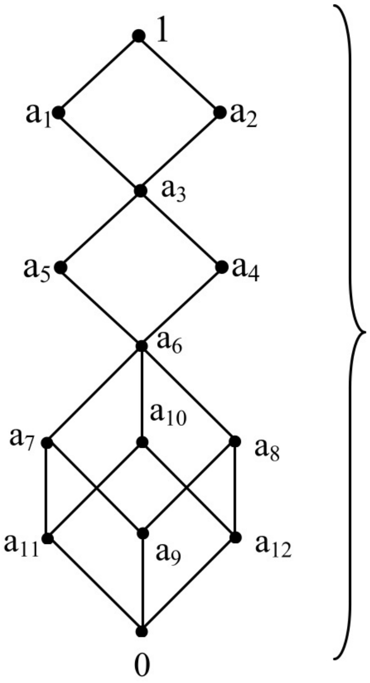
11. Let S = {Collection of all subsets from the semiring L =



    be the subset semiring of type II of finite order.

    Study questions (i) to (v) of problem (4) for this S.

12. Let S = {Collection of all subsets from the semiring M =



be the subsetsemiring of type II.

(i) Find o(S).
(ii) Find the number of subset zero divisors in S.
(iii) Find the number of subset idempotents of S.
(iv) Find the number of subset subsemirings of S.
(v) Find the number of subset semiring ideals in S.
(vi) Find the number of subset subsemirings which are not subset semiring ideals.
(vii) Is S a Smarandache subset semiring?
(viii) Can S have subset semiring S-ideals?
(ix) Can S have S-subset subsemirings?
(x) Can S have S-subset zero divisors?

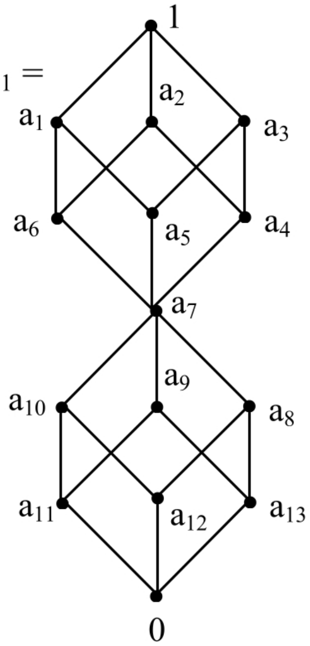
13. Study the special features enjoyed by non commutative finite subset semirings.

14. Let S = {Collection of all subsets from the semiring B, a Boolean algebra of order 32} be the subset semiring.

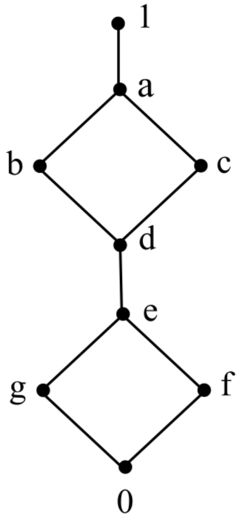
Study questions (i) to (x) of problem (12) for this S.

15. Let S = {Collection of all subsets from the semiring $L_1 \times L_2$ where

$L_1 =$



and $L_2 =$

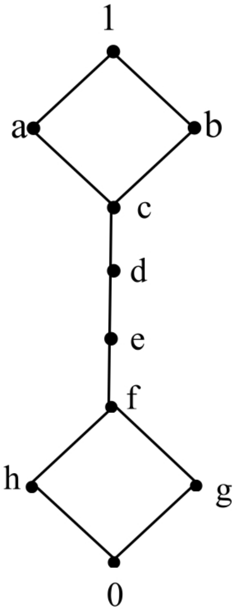


be the subsemiring of finite order.

Study questions (i) to (x) of problem (12) for this S.

16. Let S = {Collection of all subsets from the semiring $L(g) = \{a + bg \mid a, b \in L$ where

$L =$

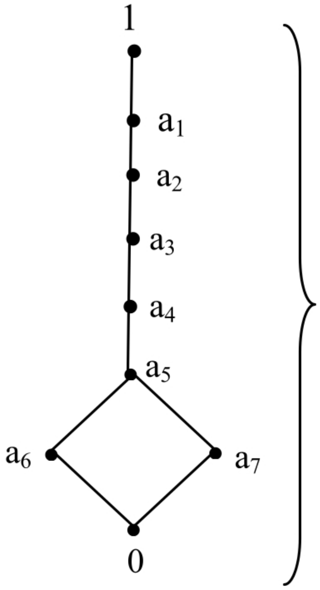


and $g^2 = 0\}\}$

be the subset semiring of type II.

Study questions (i) to (x) of problem (12) for this S.

17. Let S = {Collection of all subsets from the semiring $L(g_1,g_2) = \{a + bg_1 + cg_2 \mid a, b, c \in L,\ g_1^2 = 0,\ g_2^2 = g_2,$ $g_1g_2 = g_2g_1 = 0\}$ and L =

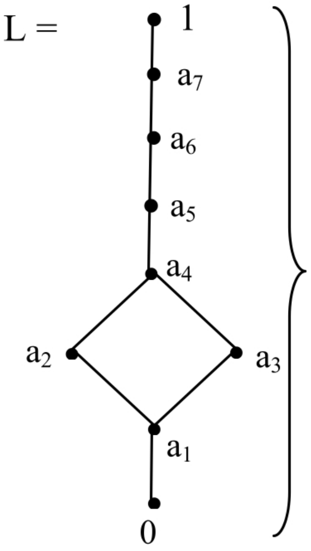


be the subset semiring of type II.

Study questions (i) to (x) of problem (12) for this S.

18. Let S = {Collection of all subsets from the semiring LA_4

where L =

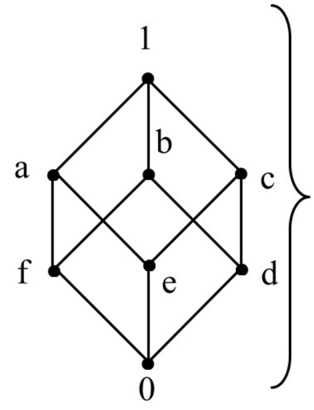


be the subset semiring of type II.

Study questions (i) to (x) of problem (12) for this S.

19. Let S = {Collection of all subsets from the semiring

LS$_3$ where L =



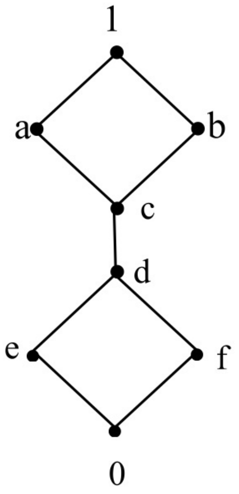
be the subset semiring of type II.

Study questions (i) to (x) of problem (12) for this S.

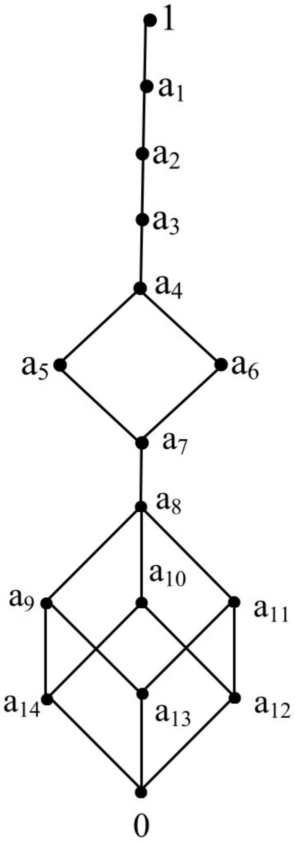
20. Let S = {Collection of all subsets from the semiring LA$_5$ where L is a Boolean algebra of order 64} be the subset semiring.

Study questions (i) to (x) of problem (12) for this S.

21. Let S = {Collection of all subsets from the semiring (L × L$_1$)S$_4$; L =



and L_1 =



be the subset semiring of type II.

Study questions (i) to (x) of problem (12) for this S.

22. Let S = {Collection of all subsets of the semiring $\langle Z^+ \cup I\rangle S(3)$} be the subset semiring of type II.

    Study questions (i) to (v) of problem (4) for this S.

    (i) Prove S is non commutative.
    (ii) Can S have subset semiring right ideals which are not subset semiring left ideals?
    (iii) Can S have right subset zero divisors which are not left subset zero divisors?

23. Let S = {Collection of all subsets from the semiring $\langle Z^+ \cup I \cup \{0\}\rangle(A_4 \times D_{27})$} be the subset semiring of type II.

Study questions (i) to (v) of problem (4) for this S.

Study questions (i) and (iii) of problem 22 for this S.

24. Let S = {Collection of all subsets from the semiring $\langle Q^+ (g_1, g_2, g_3) \cup \{0\}\rangle\}$ where $g_1^2 = 0$, $g_2^2 = g_2$, $g_3^2 = 0$, $g_ig_j = g_jg_i = 0$, $1 \leq i, j \leq 3\}$ be the subset semiring of type II.

Study questions (i) to (v) of problem (4) for this S.

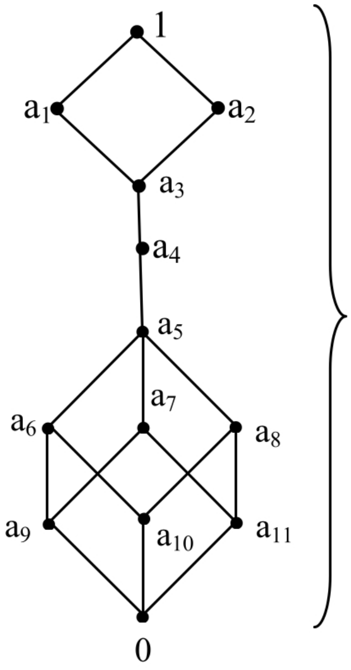
Study questions (i) to (iii) of problem (22) for this S.

25. Let S = {Collection of all subsets from the semiring $L(S_4 \times D_{29})$ where L = {Boolean algebra of order 16}} be the subset semiring of type II.

Study questions (i) to (v) of problem (4) for this S.

Study questions (i) to (iii) of problem (22) for this S.

26. Let S = {Collection of all subsets from the semiring $(L_1 \times L_2)$ $(A_4 \times D_{211})$ where L_1 is a Boolean algebra of order 32 and L_2 =



be the semiring of type II.

Study questions (i) to (v) of problem (4) for this S.

Study questions (i) to (iii) of problem (22) for this S.

27. Let S = {Collection of all subsets of the semiring $(Z^+ \cup \{0\}) D_{2,5}$} be the subset semiring of type II.

Study questions (i) to (v) of problem (4) for this S.

Study questions (i) to (iii) of problem (22) for this S.

28. Let S = {Collection of all subsets from the semiring $\langle Q^+ \cup I \cup \{0\}\rangle D_{2,13}$} be the subset semiring of type II.

Study questions (i) to (v) of problem (4) for this S.

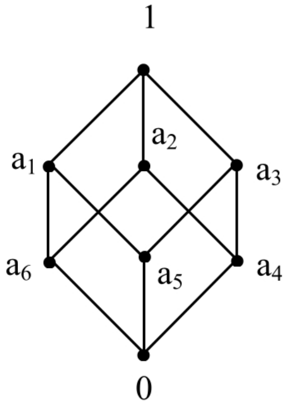
Study questions (i) to (iii) of problem (22) for this S.

29. Let S = {Collection of all subsets from the matrix semiring $M = \{(a_1, a_2, a_3, a_4, a_5) \mid a_i \in \langle Z^+ \cup I \cup \{0\}\rangle, 1 \leq i \leq 5\}$} be the subset semiring of type II.

Study questions (i) to (v) of problem (4) for this S.

Study questions (i) to (iii) of problem (22) for this S.

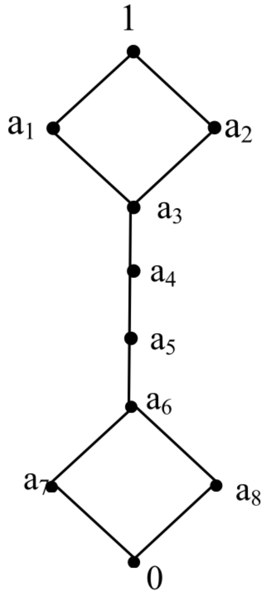
30. Let S = {Collection of all subsets from the semiring $M = \{(a_1, a_2, \ldots, a_{10}) \mid a_i \in L =$



$1 \leq i \leq 10\}$ be the subset semiring of type II.

(i) Find o(S).
(ii) Find all subset zero divisors of S.
(iii) Find all subset units of S.
(iv) Find all subset idempotents of S.
(v) Find all subset matrix subsemiring of S.
(vi) Find all subset semiring ideals of S.
(vii) Find all subset matrix subsemirings of S which are not subset matrix semiring ideals of S.
(viii) Is S a Smarandache subset matrix semiring?

31. Let S = {Collection of all subsets from the matrix semiring $M = (d_1, d_2, \ldots, d_{10}) \mid d_i \in L =$



$1 \leq i \leq 10\}\}$ be the subset semiring.

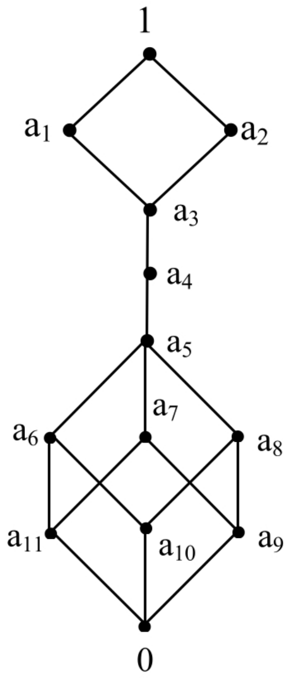
Study questions (i) to (viii) of problem 30 for this S.

32. Let S = {Collection of all subsets from the matrix

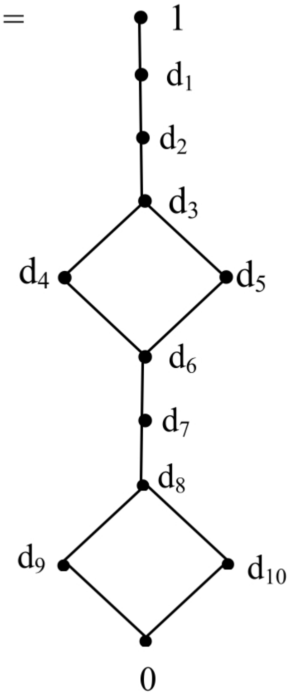
$$\text{semiring } M = \left\{ \begin{pmatrix} a_1 & a_2 & \ldots & a_{10} \\ a_{11} & a_{12} & \ldots & a_{20} \\ a_{21} & a_{22} & \ldots & a_{30} \end{pmatrix} \middle| \, a_i = (x_i, y_i) \in \text{where} \right.$$

$x_i \in L_1$ and $y_i \in L_2$ and $L = L_1 \times L_2$ with

$L_1 =$



and $L_2 =$



$1 \leq i \leq 30\}$

be the subset semiring.

Study questions (i) to (viii) of problem 30 for this S.

33. Let S = {Collection of all subsets from the matrix

$$\text{semiring } M = \left\{ \left. \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{20} \end{bmatrix} \right| a_i \in B \text{ where B is a Boolean} \right.$$

algebra of order 2^5; $1 \leq i \leq 20$} be the subset semiring.

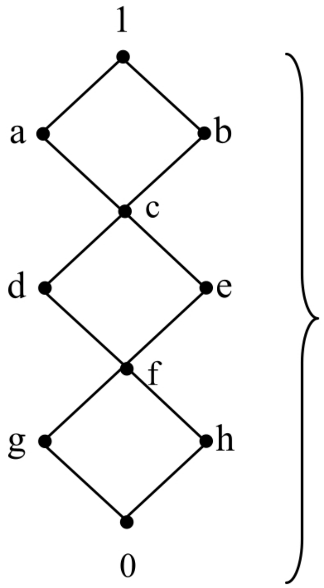
Study questions (i) to (viii) of problem 30 for this S.

34. If in problem if 33 B is replaced by a chain lattice C_{10}.

Study questions (i) to (viii) of problem 30 for this subset matrix semiring.

35. Let S = Collection of all subsets from the matrix semiring

$$M = \left\{ \left. \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \right| a_i \in LS_4;\ 1 \leq i \leq 4; \text{ and } L = \right.$$



be the subset matrix semiring.

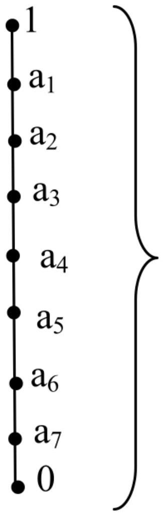
Study questions (i) to (viii) of problem (30) for this S.

36. Let S = {Collection of all subsets from the matrix

$$\text{semiring } M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{13} & a_{14} & a_{15} \end{bmatrix} \middle| \; a_i \in (L_1 \times L_2)D_{27}; \right.$$

$1 \leq i \leq 15$ where L_1 is a Boolean algebra of order 2^6 and

$L_2 = C_9 =$



be the subset matrix semiring.

Study questions (i) to (viii) of problem (30) for this S.

37. Let S = {Collection of all subsets from the matrix

$$\text{semiring } M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \middle| \; a_i \in L(D_{27} \times S_3); \right.$$

$1 \leq i \leq 16\}$ be the subset matrix semiring.

Study questions (i) to (viii) of problem 30 for this S.

38. Let S = {Collection of all subsets from the matrix

$$\text{semiring } M = \left\{ \left. \begin{pmatrix} a_1 & a_2 & \ldots & a_9 \\ a_{10} & a_{11} & \ldots & a_{18} \\ a_{19} & a_{20} & \ldots & a_{27} \end{pmatrix} \right| a_i \in \langle Z^+ \cup I \rangle\, S_3, \right.$$

$1 \leq i \leq 27$} be the subset matrix semiring of infinite order.

(i) Prove S is non commutative.
(ii) Can S have subset right zero divisors which are not subset left zero divisors?
(iii) Can S have subset idempotents?
(iv) Does S contain subset left semiring ideals which are not subset right semiring ideals and vice versa.
(v) Is S a Smarandache subset semiring?
(vi) Does S contain S-subset semiring ideals?

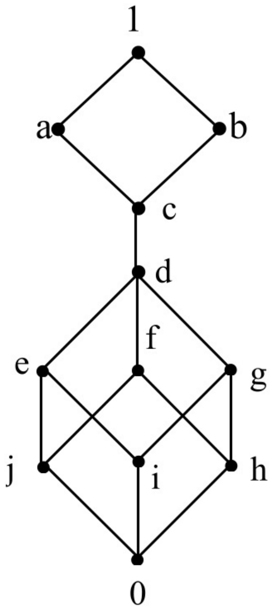
39. Give some special properties enjoyed by subset interval matrix semirings.

40. Let S = {Collection of all subsets from the interval matrix semiring M = {($[a_1, b_1]$, $[a_2, b_2]$) | a_i, $b_i \in Z^+ \cup \{0\}$, $1 \leq i \leq 2$}} be the subset interval matrix semiring.

(i) Characterize those subset zero divisors.
(ii) Find all subsets matrix interval semiring ideals of S.
(iii) Find all subset matrix interval subsemirings of S which are not subset semiring ideals of S.
(iv) Is S a Smarandache subset interval matrix semiring?
(v) Find all subset matrix subset interval zero divisors of S.

41. Let S = {Collection of all subsets from the interval row matrix semiring M = {($[a_1, b_1]$ $[a_2, b_2]$, …, $[a_6, b_6]$) | a_i, b_i are in

L =



$1 \leq i \leq 6\}$ be the subset interval row matrix semiring.

(i) Find o(S).
(ii) Study questions (i) to (v) of problem 40 for this S.

42. Let S = {Collection of all subsets from the interval row matrix semiring $M = \{([a_1, b_1] [a_2, b_2], \ldots, [a_9, b_9]) \mid a_i, b_i \in LS_3$ where L is the chain lattice C_{12}, $1 \leq i \leq 9\}$ be the subset interval row matrix semiring.

(i) Find o(S).
(ii) Prove S is non commutative.
(iii) Study questions (i) to (v) of problem 40 for this S.

43. Let S = {Collection of all subsets from the interval row matrix semiring $M = \{([a_1, b_1] [a_2, b_2], \ldots, [a_5, b_5]) \mid a_i, b_i \in (Z^+ \cup \{0\}) (g_1, g_2)$ with $g_1^2 = 0$, $g_2^2 = g_2$, $g_1g_2 = g_2g_1 = 0$, $1 \leq i \leq 5\}\}$ be the subset interval row matrix semiring.

(i) Show S is non commutative.
(ii) Study questions (i) to (v) of problem 40 for this S.

44. Let S = {Collection of all subsets from the interval row

matrix semiring M = $\left\{\begin{bmatrix}[a_1,b_1]\\ [a_2,b_2]\\ [a_3,b_3]\\ [a_4,b_4]\\ [a_5,b_5]\end{bmatrix} \middle|\ a_i, b_i \in R^+ \cup \{0\},\right.$

$1 \leq i \leq 5\}\}$ be the subset interval matrix semiring.

Study questions (i) to (v) of problem 40 for this S.

45. Let S = {Collection of all subsets from the column interval matrix semiring M = {9 × 1 interval column matrices with entries from $(Q^+ \cup \{0\})\ D_{2,9}\}\}$ be the subset interval column matrix semiring.

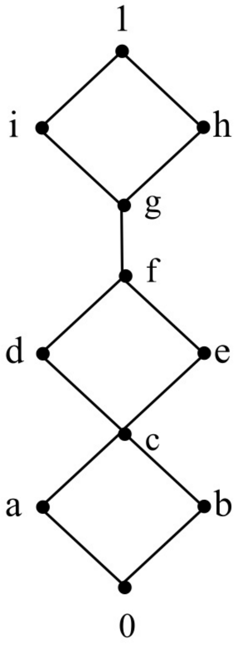
(i) Prove S is non commutative.

(ii) Study questions (i) to (v) of problem 40 for this S.

46. Let S = {Collection of all subsets from the interval

column matrix semiring M = $\left\{\begin{bmatrix}[a_1,b_1]\\ [a_2,b_2]\\ \vdots\\ [a_{15},b_{15}]\end{bmatrix} \middle|\ a_i, b_i \in\right.$

L =



1 ≤ i ≤ 15}} be the subset interval column matrix semiring.

(i) Find o(S).
(ii) Study questions (i) to (v) of problem 40 for this S.

47. Let S = {Collection of all subsets from the interval

$$\text{column matrix semiring } M = \left\{ \left. \begin{bmatrix} [a_1, b_1] \\ [a_2, b_2] \\ \vdots \\ [a_{10}, b_{10}] \end{bmatrix} \right| a_i, b_i \in LS_4 \right.$$

where L is a Boolean algebra of order 2^5; $1 \leq i \leq 10$}} be the subset interval column matrix semiring.

(i) Find o(S).
(ii) Study questions (i) to (v) of problem 40 for this S.
(iii) Prove S is non commutative.

48. Let S = {Collection of all subsets from the interval

column matrix semiring $M = \left\{\begin{bmatrix} [a_1,b_1] & [a_2,b_2] \\ [a_3,b_3] & [a_4,b_4] \\ \vdots & \vdots \\ [a_{11},b_{11}] & [a_{12},b_{12}] \end{bmatrix}\right|$

a_i, $b_i \in R^+ \cup \{0\}$, $1 \leq i \leq 12\}\}$ be the subset interval matrix semiring.

Study questions (i) to (v) of problem 40 for this S.

49. Let S = {Collection of all subsets from the interval column matrix semiring

$$M = \left\{\begin{bmatrix} [a_1,b_1] & [a_2,b_2] & \dots & [a_9,b_9] \\ [a_{10},b_{10}] & [a_{11},b_{11}] & \dots & [a_{18},b_{18}] \\ [a_{19},b_{19}] & [a_{20},b_{20}] & \dots & [a_{30},b_{30}] \end{bmatrix}\right| a_i, b_i \in L;$$

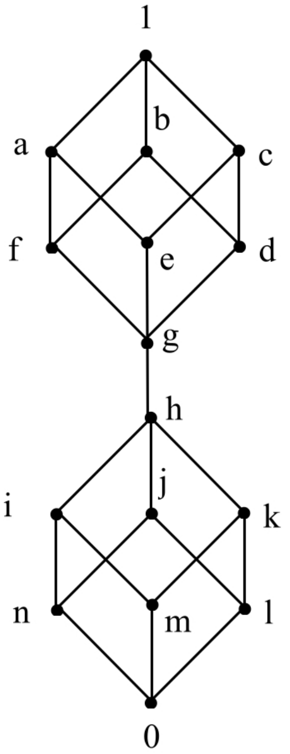
L a Boolean algebra of order 64, $1 \leq i \leq 30\}\}$ be the subset interval matrix semiring.

(i) Find o(S).
(ii) Study questions (i) to (v) of problem 40 for this S.

50. Let S = {Collection of all subsets from the interval column matrix semiring

$$M=\left\{\begin{bmatrix} [a_1,b_1] & [a_2,b_2] & [a_3,b_3] & [a_4,b_4] & [a_5,b_5] \\ [a_6,b_6] & [a_7,b_7] & [a_8,b_8] & [a_9,b_9] & [a_{10},b_{10}] \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ [a_{21},b_{21}] & [a_{22},b_{22}] & [a_{23},b_{23}] & [a_{24},b_{24}] & [a_{25},b_{25}] \end{bmatrix}\right|$$

$a_i, b_i \in L =$



$1 \leq i \leq 25\}\}$ be the subset interval matrix semiring.

(i) Find o(S).
(ii) Study questions (i) to (v) of problem 40 for this S.

51. Let S = {Collection of all subsets from the interval

$$\text{column matrix semiring } M = \left\{ \begin{bmatrix} [a_1, b_1] & \ldots & [a_6, b_6] \\ [a_7, b_7] & \ldots & [a_{12}, b_{12}] \\ \vdots & & \vdots \\ [a_{31}, b_{31}] & \ldots & [a_{36}, b_{36}] \end{bmatrix} \right|$$

$a_i, b_i \in L(D_{29} \times A_4)$ where L is a Boolean algebra of order 128, $1 \leq i \leq 36\}\}$ be the subset interval matrix semiring.

(i) Find o(S).
(ii) Study questions (i) to (v) of problem 40 for this S.

52. Let S = {Collection of all subsets from the interval

column matrix semiring M = $\left\{\left.\begin{bmatrix} [a_1, b_1] & \dots & [a_{10}, b_{10}] \\ [a_{11}, b_{11}] & \dots & [a_{20}, b_{20}] \\ \vdots & & \vdots \\ [a_{61}, b_{61}] & \dots & [a_{70}, b_{70}] \end{bmatrix}\right|\right.$

$a_i, b_i \in (Q^+ \cup \{0\})S_4$, $1 \leq i \leq 70\}\}$ be the subset interval matrix semiring.

Study questions (i) to (v) of problem 40 for this S.

53. Give some special and interesting features enjoyed by subset interval polynomial semirings of type II.

54. Distinguish type I and type II subset interval polynomial semirings.

55. Give an example of a subset interval polynomial semiring of finite order of type II.

56. Let S = {Collection of all subsets from the interval

polynomial semiring M = $\left\{\left.\sum_{i=0}^{\infty} [a_i, b_i] x^i \right| a_i, b_i \in Z^+ \cup \{0\}\right\}\}$

be the subset interval polynomial semiring of type II.

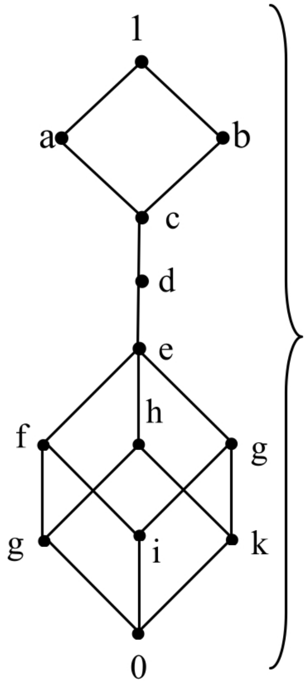
(i) Find subset interval zero divisors of S.
(ii) Find subset interval idempotents if any in S.
(iii) Find all subset interval polynomial subsemirings which are not subset interval polynomial semiring ideals of S.
(iv) Is S a Smarandache subset interval polynomial semiring?
(v) Find the collection of all subset interval annihilator ideals of S.

57. Let S = {Collection of all subsets from the interval polynomial semiring $M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i]x^i \right|$ $a_i, b_i \in (Q^+ \cup \{0\})\, S_4\}$ be the subset interval polynomial semiring.

    (i) Study questions (i) to (v) of problem 56 for this S.
    (ii) Prove S is a non commutative interval polynomial semiring.

58. Let S = {Collection of all subsets from the interval polynomial semiring

    $M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i]x^i \right|$ $a_i, b_i \in L$ where L is as follows:



the subset interval polynomial semiring.

    (i) Study questions (i) to (v) of problem 56 for this S.

59. Let S = {Collection of all subsets from the interval polynomial semiring

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in LD_{2,7} \text{ where L is a Boolean} \right.$$

algebra of order 128}} be the subset interval polynomial semiring.

(i) Prove S is a non commutative.
(ii) Study questions (i) to (v) of problem 56 for this S.

60. Let S = {Collection of all subsets from the interval polynomial semiring

$$M = \left\{ \sum_{i=0}^{9} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in (Z^+ \cup \{0\}), S_7, 0 \le i \le 9 \text{ and } x^{10} = 1 \right\}\}$$

be the subset interval polynomial semiring.

Study questions (i) to (v) of problem 56 for this S.

61. Let S = {Collection of all subsets from the interval polynomial semiring

$$M = \left\{ \sum_{i=0}^{9} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in L, 0 \le i \le 8, x^9 = 1 \right.$$

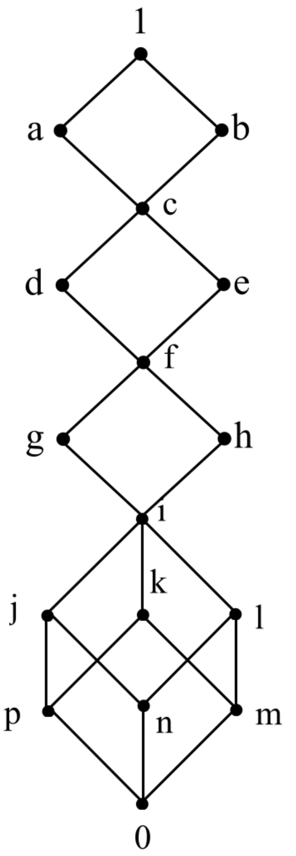
where L is a chain lattice C_{25}}} be the subset interval polynomial semiring.

(i) Find o(S).
(ii) Study questions (i) to (v) of problem 56 for this S.

62. Let S = {Collection of all subsets from the interval polynomial semiring

$$M = \left\{ \sum_{i=0}^{9} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in LD_{211}, 0 \leq i \leq 5, x^6 = 1; \right.$$

L =



be the subset interval polynomial semiring.

(i) Find o(S).
(ii) Prove S is non commutative.
(iii) Study questions (i) to (v) of problem 56 for this S.

63. Enumerate some special features enjoyed by non commutative subset interval polynomial semirings of finite order.

64. Give an example of a non commutative subset interval polynomial semiring which has left subset zero divisors which are not right subset zero divisors.

65. Characterize those subset interval polynomial semirings which has Smarandache subset zero divisors?

66. Can a subset interval polynomial semiring have S-subset interval units?

67. Can a subset interval polynomial semiring have Smarandache subset idempotents?

68. Describe any other stricking features enjoyed by subset interval polynomial semirings.

69. Characterize those subset interval polynomial semirings in which every subset interval polynomial semiring ideal is Smarandache.

70. Characterize those subset interval polynomial semiring in which no subset interval polynomial semiring ideal is Smarandache.

71. Characterize those subset interval polynomial semiring in which every subset interval polynomial subsemiring is Smarandache.

72. Characterize those subset interval polynomial semiring S in which no subset interval polynomial subsemiring is Smarandache but S is a Smarandache subset interval polynomial semiring.

73. Does there exist a subset interval polynomial semiring which is not Smarandache?

74. Can there be a subset interval polynomial semiring which is a subset interval semifield?

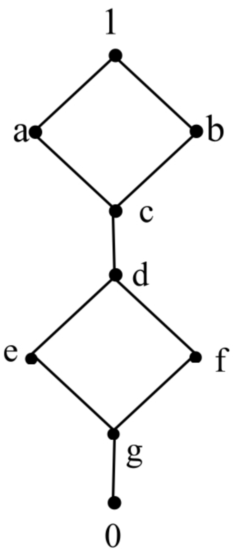
75. Does there exists an infinite subset interval polynomial semiring which has no subset interval polynomial semiring ideals?

76. Obtain some special features enjoyed by S = {Collection of all subsets from the interval polynomial semiring

$$M = \left\{ \sum_{i=0}^{\infty} [a_i, b_i] x^i \,\middle|\, a_i, b_i \in \langle Z^+ \cup \{0\} \cup I \rangle \right\}\}.$$

77. If in problem 76; $\langle Z^+ \cup \{0\} \cup I \rangle$ is replaced by $\langle R^+ \cup \{0\} \cup I \rangle$ study the special features and distinguish it from S in problem 76.

78. If in problem 76, $\langle Z^+ \cup \{0\} \cup I \rangle$ is replaced by $L_1 \times L_2 \times L_3$ where L = C_{20} a chain lattice, L_2 a Boolean algebra of order 32 and L_3 =



Study this and compare it with problems 76 and 77.

79. If in problem 77, $\langle Z^+ \cup \{0\} \cup I \rangle$ is replaced by $\langle Z^+ \cup \{0\} \cup I \rangle (S_3 \times D_{211})$.

Study the problem and compare it with problems 77 and 78.

80. Find some innovative and interesting applications of subset semiring interval polynomials of finite order.

Chapter Four

NEW SUBSET SPECIAL TYPE OF TOPOLOGICAL SPACES

In this chapter we just study and briefly introduce different types of subset topological special type of semiring spaces associated with them. We give atleast four types of topological spaces associated with type I or type II subset semirings.

DEFINITION 4.1: *Let S be a subset semiring of type I.* $T_o = \{S', \cup, \cap; S' = S \cup \{\phi\}\}$ *is the ordinary subset topological semiring space of type I of S.*

$T_\cup = \{S' = S \cup \{\phi\}, \cup, \times\}$ *is a new type of subset special topological semiring space of type I of S.*

$T_\cap = \{S' = S \cup \{\phi\}, +, \cap\}$ *is a new type of subset special topological semiring type I space of S.*

$T_s = \{S, +, \times\}$ *is defined as the new type of special topological semiring type I with inherited operations of the ring or semiring space of S.*

We see for a given subset semiring S of type I we can associate four different types of topological semiring type I spaces of S.

We will illustrate this by some examples.

Example 4.1: Let
S = {Collection of all subsets from the ring Z_{12}} be the type I subset semiring.

We will give all the four types of topological spaces for this type I subset semiring. Let T_o, $T_\cup$, $T_\cap$ and T_s be the four types of special subset topological type I semiring spaces.

Let A = {2, 11, 8} B = {5, 7, 0, 2} $\in S' = S \cup \{\phi\} \in T_o$.

We find

$$\begin{aligned} &A \cup B \\ &\quad = \{2, 11, 8\} \cup \{5, 7, 0, 2\} \\ &\quad = \{0, 2, 5, 7, 8, 11\} \qquad \ldots \quad I \end{aligned}$$

and

$$\begin{aligned} &A \cap B \\ &\quad = \{2, 11, 8\} \cap \{5, 7, 0, 2\} \\ &\quad = \{2\} \text{ are in } T_o. \end{aligned}$$

Let A, B $\in T_\cup$ we see $A \cup B$ is I

$$\begin{aligned} &A \times B \\ &\quad = \{2, 11, 8\} \times \{5, 7, 0, 2\} \\ &\quad = \{0, 4, 10, 2, 5, 7, 8\}. \end{aligned}$$

We see $T_\cup$ and T_o are different as topological spaces.

Consider A, B $\in T_\cap$

$$\begin{aligned} &A + B \\ &\quad = \{2, 11, 8\} + \{0, 2, 5, 7\} \\ &\quad = \{2, 11, 8, 4, 1, 10, 7, 9, 6, 3\}. \end{aligned}$$

$A \cap B$

$= \{2, 11, 8\} \cap \{5, 7, 0, 2\}$

$= \{2\}$.

$A + B$ and $A \cap B$ are in $T_\cap$.

So $T_\cup$, T_o are also different from $T_\cap$ as topological spaces.

We now study for A, B in T_s.

$A + B$

$= \{2, 11, 8\} + \{0, 2, 5, 7\}$

$= \{2, 11, 8, 4, 1, 10, 7, 9, 6, 3\}$ and

$A \times B$

$= \{2, 11, 8\} \times \{0, 2, 5, 7\}$

$= \{0, 4, 10, 7, 2, 5, 8\}$ are in T_s.

T_s is different from the topological spaces T_o, $T_\cup$ and $T_\cap$.

Thus all the four types of special new topological semiring spaces of type I are described.

Example 4.2: Let
S = {Collection of all subsets from the ring of integers Z} be the subset semiring of type I.

Let $A = \{-8, 6, 0, 9, 12, -3\}$
and $B = \{6, 5, 1, -1, 4, -7\} \in S$

Let T_o be the ordinary topological space of type I.

Consider

$A \cup B = \{-8, 6, 0, 9, 12, -3\} + \{6, 5, 1, -1, 4, -7\}$

$= \{0, 1, -1, 4, -7, 6, 5, 9, 8, 12, -3\}$ and

$$\begin{aligned} A \cap B &= \{-8, 6, 9, 0, 12, -3\} \cap \{6, 5, 1, -1, 4, -7\} \\ &= \{6\}. \end{aligned}$$

Clearly $A \cup B$ and $A \cap B$ are in T_o.

Consider $T_\cup$, the special new type of topological semiring space of type I.

$$\begin{aligned} A \cup B &= \{-8, 6, 0, 9, 12, -3\} \cup \{6, 5, 1, -1, 4, -7\} \\ &= \{0, 1, -1, 4, -7, 6, 5, 9, -8, 12, -3\} \text{ and} \end{aligned}$$

$$\begin{aligned} A \times B &= \{0, -8, 6, 9, 12, -3\} \times \{6, 5, 1, -1, 4, -7\} \\ &= \{0, -48, 36, 54, 72, -18, -40, 30, 45, 60, -15, -8, \\ &\quad 6, 9, 12, -3, 8, -6, -9, -12, 3, -32, 24, 48, 56, \\ &\quad -42, -63, -72, 21\}, \end{aligned}$$

both $A \cup B$ and $A \times B$ are in $T_\cup$.

We see $T_\cup$ and T_o are distinctly different as topological spaces.

Consider $A, B \in T_\cap$;

$$\begin{aligned} A + B &= \{0, -8, 6, 9, 12, -3\} + \{6, 5, 1, -1, 4, -7\} \\ &= \{6, 5, 1, -1, 4, -7, -2, -3, -7, -9, -4, -15, 12, 11, \\ &\quad 7, 5, 10, -1, 15, 14, 10, 8, 13, 2, 18, 17, 13, 11, \\ &\quad 16, 5, 3, 2, -2, -4, 1, -10\} \\ &= \{6, 5, 1, -1, 4, -7, -2, -9, -4, -15, 12, 11, 7, 10, \\ &\quad 15, 14, 10, 8, 13, 2, 18, 17, 16, 3, -10\} \end{aligned}$$

and

$$A \cap B = \{-8, 6, 9, 0, 12, -3\} \cap \{6, 5, 1, -1, 4, -7\} = \{6\}.$$

$A + B$ and $A \cap B$ are in $T_\cap$.

We see $T_\cup$, $T_\cap$ and T_o are different as topological spaces.

Let A, B $\in T_s$.

$A + B = \{0, -8, 6, 9, 12, -3\} + \{6, 5, 1, -1, 4, -7\}$

$= \{6, 5, 1, -1, 4, -7, -2, 12, 15, 18, 3, 11, 14, 17, 2, -7, 7, 10, 13, -2, -9, 5, 8, 11, -4, 10, 13, 16, 1, -15, -1, 2, 5, -10\}$ and

$= \{6, 5, 1, -1, 4, -7, -2, 12, 15, 18, 3, -3, 11, -10, 14, 17, 2, 7, 10, 13, -9, 5, 8, -4, 16, -15\}$ and

$A \times B = \{0, -8, 6, 9, 12, -3\} \times \{6, 5, 1, -1, 4, -7\}$

$= \{0, -48, 36, 54, 72, -18, -40, 30, 45, 60, -15, -8, 6, 9, 12, -3, 3, -12, -9, -6, 8, -32, 24, 36, 48, -12, 56, -42, -63, -84, 21\}$

$= \{0, -48, 36, 54, 72, -18, -40, 30, 45, 60, -15, -8, 6, 9, 12, -3, 3, -12, -9, -6, 8, -32, 24, 48, 56, -42, -63, -84, 21\}$ are in T_s.

T_s is different from T_o, $T_\cup$ and $T_\cap$ as topological spaces.

Example 4.3: Let
S = {Collection of all subsets from the ring $C(Z_6)$} be the subset semiring of type I. Let T_o, $T_\cup$, $T_\cap$ and T_s be the four new types of special subset topological semiring spaces of type I.

$A = \{i_F, 2 + 3i_F, 0, 4+5i_F\}$ and
$B = \{2 + 3i_F, 0, 4, 3i_F, 5 + 2i_F, 2 + 2i_F\} \in S$

Let us take A, B $\in T_o$ the ordinary special topological semiring space of type I of S.

$A \cup B = \{i_F, 2 + 3i_F, 0, 4 + 5i_F\} \cup \{2 + 3i_F, 0, 4, 3i_F, 5 + 2i_F, 2+2i_F\}$

$= \{i_F, 2+3i_F, 0, 4+5i_F, 4, 3i_F, 5+2i_F, 2+2i_F\}$ and

$A \cap B = \{i_F, 2+3i_F, 0, 4+5i_F\} \cap \{0, 4, 3i_F, 2+3i_F, 5+2i_F, 2+2i_F\}$

$= \{0, 2+3i_F\}$.

$A \cap B, A \cup B \in T_o$.

Now let $A, B \in T_\cup$

To find $A \cup B$ and $A \times B$.

We see

$A \cup B = \{i_F, 2+3i_F, 0, 4+5i_F\} \cup \{0, 4, 2+3i_F, 3i_F, 5+2i_F, 2+2i_F\}$

$= \{i_F, 2+3i_F, 4+5i_F, 0, 4, 3i_F, 2+2i_F, 5+2i_F\}$ and

$A \times B = \{i_F, 2+3i_F, 0, 4+5i_F\} \times \{0, 4, 2+3i_F, 3i_F, 5+2i_F, 2+2i_F\}$

$= \{0, 4i_F, 2, 4+2i_F, 2i_F+3, 1, 5+4i_F, 3, 5i_F+4, 4+i_F, 4+3i_F, 2i_F+4, 4+4i_F, 2\}$ are in $T_\cup$.

$T_\cup$ is a special new topological semiring type I space different from T_o.

Let $A, B \in T_\cap$.

$A \cap B = \{i_F, 2+3i_F, 0, 4+5i_F\} \cap \{2+3i_F, 0, 4, 3i_F, 5+2i_F, 2+2i_F\}$

$= \{0, 2 + 3i_F\}$ and

$A + B = \{i_F, 2+3i_F, 0, 4+5i_F\} + \{2 + 3i_F, 0, 4, 3i_F, 5+2i_F, 2+2i_F\}$

$= \{2+4i_F, 4+i_F, 2+3i_F, 2i_F, 0, i_F, 4+5i_F, 3i_F, 4, 2+5i_F, 4i_F, 2, 4+2i_F, 5+3i_F, 1+5i_F, 5+2i_F, 3+i_F, 2+3i_F, 2+3i_F, 2+2i_F\} \in T_\cap.$

We see $T_\cap$ is different from T_o and $T_\cup$ as topological spaces.

Now we see if A, B $\in T_s$.

$A + B = \{i_F, 3i_F + 2, 0, 4+5i_F\} + \{0, 4, 3i_F, 2+3i_F, 5+2i_F, 2+2i_F\}$

$= \{0, 2+4i_F, 4+i_F, 2+3i_F, 2i_F, i_F, 4+5i_F, 3i_F, 4, 2+5i_F, 4i_F, 2, 4+2i_F, 5+3i_F, 1+5i_F, 2+2i_F, 5+2i_F, 3+i_F, 2+3i_F\}$ and

$A \times B = \{i_F, 0, 2+3i_F, 4+5i_F\} \times \{0, 4, 2+3i_F, 3i_F, 5+2i_F, 2+2i_F\}$

$= \{0, 4i_F, 2, 4+2i_F, 1, 2i_F+3, 3, 5+4i_F, 5i_F+4, 4+i_F, 4+3i_F, 2i_F+4, 4+4i_F, 2\} \in T_s.$

We see T_s is also different from all the three special type of subset topological semiring type I spaces.

Example 4.4: Let
S = {Collection of all subsets from the ring $C(\langle Z_{11} \cup I\rangle)$} be the subset semiring of type I.

Associated with S we have four distinct special topological subset semiring spaces of type I.

Example 4.5: Let
S = {Collection of all subsets from the ring $\langle C \cup I\rangle$} be the subset semiring of type I.

This also has four subset special topological semiring spaces of type I associated with it.

We have both infinite and finite subset special topological semiring spaces of type I.

We can as in case of usual spaces define set ideal (ideal) subset topological semiring spaces of type I.

Let R be a ring and I be a ideal of R.

S = {Collection of all subsets of the ring R} be the subset semiring of type I.

M = {Collection of all set ideals of R over P, P a subring of the ring R}.

S_1 = {Collection of all subsets from M} $\subseteq$ S.

S_1 is defined as the subset set ideal of S over the subring P of R.

We see S_1 can be given a topology and S_1 with a topology will be defined as the subset set ideal topological semiring of type I space over the subring P of R.

We will illustrate this situation by an example or two.

Example 4.6: Let
S = {Collection of all subsets from the ring Z_{12}} be the subset semiring of type I. Let P = {0, 4, 8} $\subseteq Z_{12}$ be the subring of S.

Now M = {Collection all subset semiring set ideals of S over the subring P of Z_{12}}. M can be given all the four topologies and it will be denoted by T_o^P, $T_\cup^P$, $T_\cap^P$ and T_S^P where T_o^P is the ordinary set subset ideal semiring topological space related to M of P; P the subring of Z_{12}.

$T_\cup^P$ is the new type subset set ideal special topological space where $T_\cup^P$ = {M, $\cup$, $\times$} related to the subring P $\subseteq Z_{12}$.

$T_{\cap}^{P} = \{M' = M \cup \{\phi\}, +, \cap\}$ related to the subring $P \subseteq Z_{12}$ is the new type subset set ideal special topological space.

$T_{S}^{P} = \{M, +, \times\}$ is the special set ideal subset topological space of the semiring S with the inherited operations of the ring related to the subring P of Z_{12}.

These definitions are a matter of routine for they are subset set ideal new type of topological spaces of the semiring S.

We will first illustrate this situation by an example or two.

Example 4.7: Let
S = {Collection of all subsets from the ring Z_6} be the subset semiring of type I of the ring Z_6. Let $P_1 = \{0, 3\} \subseteq Z_6$ be a subring of Z_6.

Now we want to find the collection of all subset set ideal of S over the subring P_1 of Z_6.

M = {{0}, {0, 1, 3}, {0, 2}, {3}, {0, 4}, {0, 2, 4}, {0, 2, 1, 3}, {0, 4, 1, 3}, {0, 2, 4, 1, 3}, {0, 3}, {0, 2, 3}, {0, 4, 3}, {0, 2, 4, 3}, {5, 3}, {0, 5, 3}, {0, 5, 3, 2}, {0, 5, 3, 4}, {1, 3}, {0, 5, 3, 2, 4}, {0, 5, 3, 2, 4, 1}, {0, 5, 3, 1}, {0, 1, 5, 3, 2}, {0, 1, 5, 3, 4}, {1, 3, 5}} ⊆ S is a collection of all subset set ideals of semiring related to the subring P_1 of Z_6.

We have $T_{o}^{P_1} = \{M', \cup, \cap\}$ to be a subset set ideal topological semiring space of type I related to the subring P_1 of Z_6.

Let A = {2, 0, 4} and B = {0, 5, 3, 4} ∈ $T_{o}^{P_1}$.
A ∪ B = {2, 0, 4} ∪ {0, 5, 3, 4} = {5, 0, 2, 3, 4} and
A ∩ B = {0, 2, 4} ∩ {0, 5, 3, 4} = {0, 4} ∈ $T_{o}^{P_1}$.

Consider $T_{\cup}^{P_1} = \{M, \cup, \times\}$, the special subset set ideal topological semiring space of type I related to $P_1 \subseteq Z_6$.

For A = {0, 2, 4} and B = {0, 5, 3, 4} $\in T_{\cup}^{P_1}$ we see

$$\begin{aligned} A \cup B &= \{0, 2, 4\} \cup \{0, 5, 3, 4\} \\ &= \{0, 2, 4, 5, 3\} \text{ and} \end{aligned}$$

$$\begin{aligned} A \times B &= \{0, 4, 2\} \times \{0, 5, 3, 4\} \\ &= \{0, 4, 2\} \in T_{\cup}^{P_1}. \end{aligned}$$

We see $T_{\cup}^{P_1}$ is different from $T_{o}^{P_1}$ as topological spaces.

Consider $T_{\cap}^{P_1}$ = {M′, +, ∩} be the special new set ideal subset topological space related to the subring P_1.

Let A = {0, 4, 2} and B = {0, 5, 3, 4};
A + B = {0, 4, 2} + {0, 5, 3, 4} = {0, 4, 2, 3, 5 1} and
A ∩ B = {0, 2, 4} ∩ {0, 5, 3, 4} = {0, 4} $\in T_{\cap}^{P_1}$.

We see $T_{\cap}^{P_1}$ is different from $T_{\cup}^{P_1}$ and $T_{o}^{P_1}$ as topological spaces.

Now finally take $T_{S}^{P_1}$ = {M, +, ×} be the subset set ideal new topological inherited semiring space related to P_1.

$$\begin{aligned} A + B &= \{0, 4, 2\} + \{0, 5, 3, 4\} \\ &= \{0, 1, 2, 3, 4, 5\} \end{aligned}$$

$$\begin{aligned} \text{and } A \times B &= \{0, 4, 2\} \times \{0, 5, 3, 4\} \\ &= \{0, 2, 4\} \in T_{S}^{P_1}. \end{aligned}$$

We see $T_{S}^{P_1}$ is distinctly different from $T_{o}^{P_1}$, $T_{\cup}^{P_1}$ and $T_{\cap}^{P_1}$ as topological spaces.

Thus we see for a given subset subsemiring of the ring we have four related subset set ideal new topological special type of semiring spaces of the semiring.

Example 4.8: Let
S = {Collection of all subsets from the ring Z(g)} be the subset semiring of type I. Let M = 2Z(g) be a subring of Z(g).

Let P = {Collection of all subsets of semiring set ideals of S over the subring M = 2Z(g)} $\subseteq$ S is such that the four new special subset set ideal topological spaces can be defined on P viz. T_o^M, T_S^M, $T_\cup^M$ and $T_\cap^M$.

All of them will be of infinite order.

Example 4.9: Let
S = {Collection of all subsets from the ring $Z_3(g)$; $g^2 = 0$} be the subset semiring of type I. Take P = Z_3 the subring of S.

M = {Collection of all subset set ideals of S over the subring Z_3 of $Z_3(g)$} be the new subset set ideal semiring topological spaces T_o^P, T_S^P, $T_\cup^P$ and $T_\cap^P$ related to the subring P = Z_3.

We just take A = {0, g, 2g} and
B = {0, 1, 2, 1+g, 2+2g} $\in$ M.

$$\begin{aligned} A + B &= \{0, g, 2g\} + \{0, 1, 2, 1+g, 2+2g\} \\ &= \{0, g+1, 2g+1, 2+g, g, 2g, 2+2g, 1+g, 1, 2, 2+g\} \\ &\quad \in M \text{ and} \end{aligned}$$

$$\begin{aligned} A \times B &= \{0, g, 2g\} \times \{0, 1, 2, 1+g, 2+2g\} \\ &= \{0, g, 2g\} \in T_S^P. \end{aligned}$$

$$\begin{aligned} A \cup B &= \{0, g, 2g\} \cup \{0, 1, 2, 1+g, 2+2g\} \\ &= \{0, 1, 2, g, 2g, 1+g, 2+2g\} \text{ and} \end{aligned}$$

$$A \cap B = \{0, g, 2g\} \cap \{0, 1, 2, 1+g, 2+2g\} = \{0\} \in T_o^P.$$

T_o^P and T_S^P are distinctly different as topological spaces.

Consider

$$\begin{aligned} A + B &= \{0, g, 2g\} + \{0, 1, 2, g+1, 2+2g\} \\ &= \{0, g+1, 2g+1, 2+g, g, 2g, 2+2g, 1+g, 1, 2, 2+g\} \end{aligned}$$

and

$$\begin{aligned} A \cap B &= \{0, 2, 2g\} \cap \{0, 1, 2, 1+g, 2+2g\} \\ &= \{0\} \in T_{\cap}^{P}. \end{aligned}$$

Thus $T_{\cap}^{P}$ is different from T_{o}^{P} and T_{S}^{P} as topological spaces.

Let

$$\begin{aligned} A \cup B &= \{0, g, 2g\} \cup \{0, 1, 2, 1+g, 2+2g\} \\ &= \{0, g, 2g, 1, 2, 1+g, 2+2g\} \end{aligned}$$

and

$$\begin{aligned} A \times B &= \{0, g, 2g\} \times \{0, 1, 2, 2+2g, 1+g\} \\ &= \{0, g, 2g\} \in T_{\cup}^{P}. \end{aligned}$$

$T_{\cup}^{P}$ is different from $T_{\cap}^{P}$, T_{o}^{P} and T_{S}^{P} new special type of set ideal semiring topological spaces of $M \subseteq S$.

We see M_1 = {Collection of all subset set semiring ideals of the subring $P_1 = \{0, g, 2g\} \subseteq Z_3(g)$} is such that $T_{o}^{P_1}$, $T_{\cup}^{P_1}$, $T_{\cap}^{P_1}$ and $T_{S}^{P_1}$ are four distinct new special subset set semiring ideal topological spaces related to the subring P_1.

This is the way new type of subset set semiring ideal finite topological spaces are constructed relative to subrings of the ring.

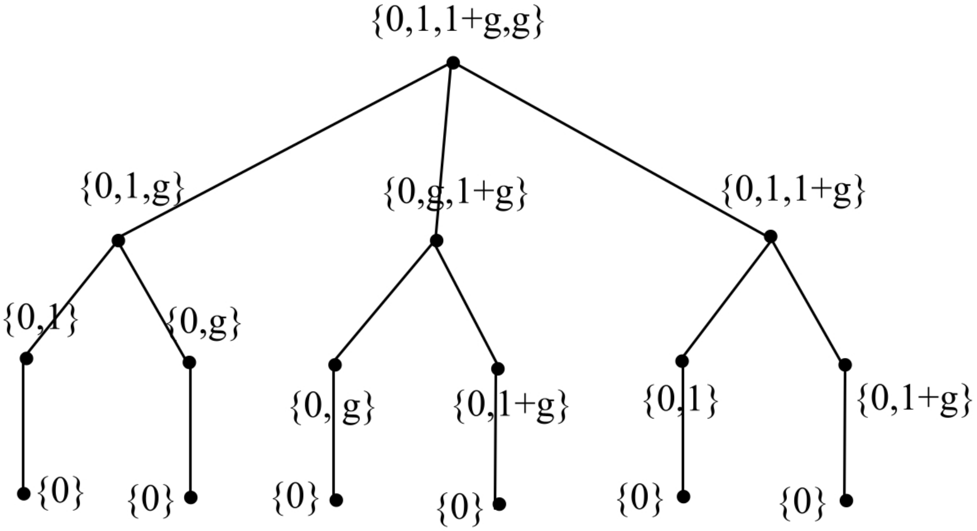
Example 4.10: Let
S = {Collection of all subsets from the ring $Z_2(g)$ where $g^2 = g$} be the subset semiring of type I. Take $P = Z_2 = \{0, 1\} \subseteq Z_2(g)$ to be a subring of $Z_2(g)$.

M = {Collection of all set ideals of the subset semiring S related to the subring Z_2}

= {{0}, {0, 1}, {0, g}, {0, 1+g}, {0, 1, g}, {0, 1, 1+g}, {0, g, 1+g}, {0, 1, 1+g, g}} is the subset semiring set ideals of S over the subset semiring. Infact each subset in M is a set ideal of S over Z_2 = {0, 1}.

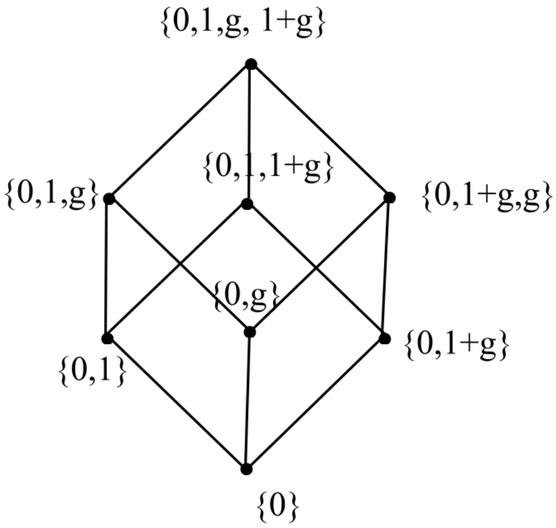
Now we see $T_o^P, T_\cup^P$, $T_\cap^P$ and T_S^P are special new subset set semiring ideal topological space of S related to P = {0, 1} = Z_2.

We see



This is the tree associated with M.

However if we wish to get the graph of M it would be entirely different.

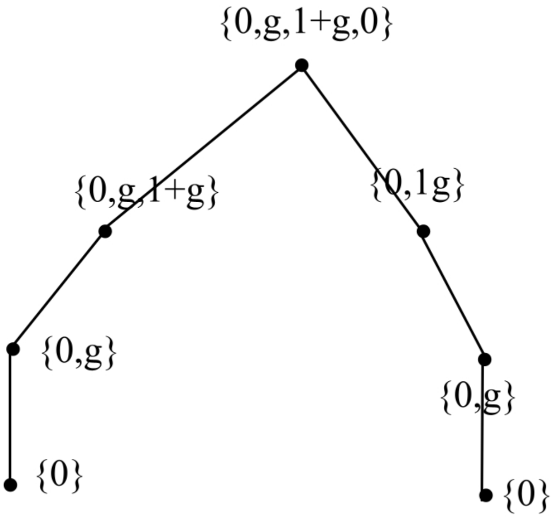


which is also a lattice, which is a Boolean algebra of order 8.

Suppose we take $P_1 = \{0, g\}$ to be the subring of $Z_2(g)$. To find the collection of all subset set semiring ideals related to P_1.

$M_1 = \{\{0\}, \{0, 1, g\}, \{0, g\}, \{0, g, 1+g\}, \{0, 1, g, 1+g\}\}$ is the new special type of subset set semiring ideal topological spaces of all the four types $T_{\cup}^{P_1}$, $T_{\cap}^{P_1}$, $T_{o}^{P_1}$ and $T_{S}^{P_1}$.

The tree associated with M_1 is

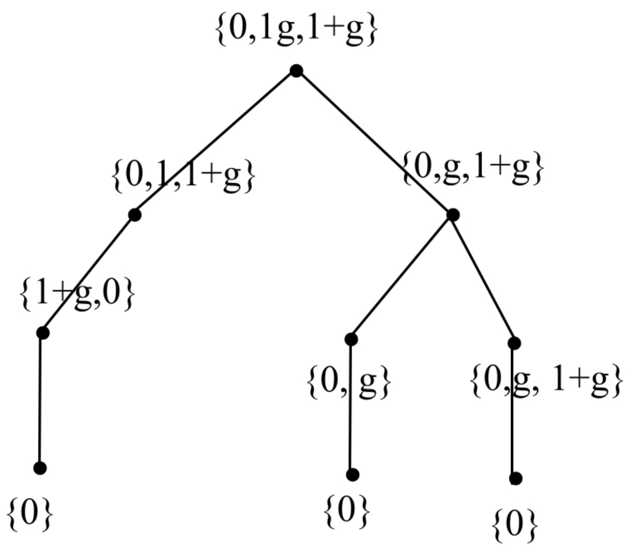


Let $P_2 = \{0, 1+g\}$ be the subring of $Z_2(g)$.

Let M_2 = {Collection of all subset set semiring ideal of S associated with P_2}.

M_2 can be given the four different new special set ideal semiring topologies.

The tree associated with $M_2 = \{\{0\}, \{0, 1+g\}, \{0, 1, 1+g\}, \{0, g\}, \{0, g, 1+g\}, \{0, g, 1, 1+g\}\}$



Example 4.11: Let
S = {Collection of all subsets from the ring $Z_4(g)$, $g^2 = 0$} be the subset semiring of type I of the ring $Z_4(g)$.

P_1 = {0, 2}, P_2 = {0, 2g}, P_3 = {0, 2 + 2g}, P_4 = {0, 2, 2g, 2+2g},

P_5 = {0, g, 2g, 3g}, P_6 = {0, 1, 2, 3} and P_7 = {0, 1+g, 2+2g, 3+3g} are some of the subrings of $Z_4(g)$.

Related with each of the subring we can build on the collection of all subset set semiring ideals special new types of topological spaces;

$T_o^{P_i}$, $T_\cup^{P_i}$, $T_\cap^{P_i}$ and $T_S^{P_i}$ for $1 \leq i \leq 7$.

M_1 = {{0}, {0, 2}, {0, 1, 2}, {0, 3, 2}, {0, 1, 3, 2}, {0, 2g}, {0, 2g, 2}, {0, 1, 2g, 2}, {0, 3, 2, 2g}, {0, 1, 3, 2, 2g}, {0, g, 2g}, {0, 3g, 2g} {0, 1, 3, 3g, 2g}…} related to the subring P_1 = {0, 2}.

M_2 = {{0}, {0, 2}, {0, 1, 2g}, {0, 2, 2g}, {0,2g}, …} related to the subring P_2 = {0, 2g}.

Let $P_5 = \{0, g, 2g, 3g\}$ be the subring of $Z_4(g)$; to find the collection of all subset set ideals of S relative to P_5.

Let M_5 = {Collection of all subset set ideals of S relative to the subring P_5} = {{0}, {0, 1, g, 2g, 3g}, {0, 2, 2g}, {0, 2g}, {0, 3, 3g, 2g}, {0, 3g, 2g}, {0, g}, {0, 3g}, {0, 1+g, g, 2g, 3g}, {0, 2+2, 2g}, {0, 3+3g, 3g, 2g}, {0, 1+2g, g, 2g, 3g}, {0, 2+g, 2g}, {0, 3+g, 3g, 2g}, {0, 3g+1, g, 2g, 3g}, …} is the collection.

Interested reader can find the trees associated with each of the M_i's; $1 \leq i \leq 7$.

Next we can proceed onto describe type II subset semirings of a semiring.

Example 4.12: Let S = {Collection of all subsets from the semiring $Z^+ \cup \{0\}$} be the subset semiring of type II over the semiring $Z^+ \cup \{0\}$.

Let T_o, $T_\cup$, $T_\cap$ and T_s be the special new type subset topological semiring spaces of type II of S.

We will show for some A, B $\in$ S; the four topological spaces T_o, $T_\cup$, $T_\cap$ and T_s are distinct.

Let A = {5, 3, 2, 0, 7} and B = {10, 8, 9, 7, 1, 4} $\in T_o$

$$\begin{aligned} A \cup B &= \{5, 2, 3, 0, 7\} \cup \{10, 8, 9, 7, 1, 4\} \\ &= \{0, 1, 2, 3, 4, 5, 7, 8, 9, 10\} \in T_o \text{ and} \end{aligned}$$

$$\begin{aligned} A \cap B &= \{5, 2, 3, 0, 7\} \cap \{10, 8, 9, 7, 1, 4\} \\ &= \{7\} \in T_o. \end{aligned}$$

Now take the same A, B $\in T_\cup$.

$$\begin{aligned} A \cup B &= \{5, 2, 3, 0, 7\} \cup \{10, 8, 9, 7, 1, 4\} \\ &= \{0, 1, 2, 3, 4, 5, 7, 8, 9, 10\} \in T_o. \end{aligned}$$

But

$$\begin{aligned} A \times B &= \{5, 2, 3, 0, 7\} \times \{10, 8, 9, 7, 1, 4\} \\ &= \{50, 0, 20, 30, 70, 40, 16, 24, 56, 45, 18, 27, \\ &\quad 63, 35, 14, 21, 49, 5, 0, 2, 3, 7, 8, 12, 28\} \in T_{\cup}. \end{aligned}$$

We see T_o and $T_{\cup}$ are different as topological spaces.

Let for the same A, B $\in T_{\cap}$ we find A + B and A $\cap$ B.

$$\begin{aligned} A + B &= \{5, 2, 3, 0, 7\} + \{10, 8, 9, 7, 1, 4\} \\ &= \{10, 8, 9, 7, 1, 4, 12, 11, 3, 6, 13, 15, 14, 17, 16\} \\ &\qquad \in T_{\cap}. \end{aligned}$$

$$A \cap B = \{5, 2, 3, 0, 7\} \cap \{10, 8, 9, 7, 1, 4\} = \{0\} \in T_{\cap}.$$

Thus $T_{\cap}$ is different from $T_{\cup}$ and T_o because of the operations defined on them.

Consider A, B $\in T_s$ we find

$$\begin{aligned} A + B &= \{0, 3, 2, 5, 7\} + \{1, 4, 8, 7, 9, 10\} \\ &= \{1, 4, 8, 7, 9, 10, 11, 12, 13, 6, 14, 15, 16, 17\} \end{aligned}$$

and
$$\begin{aligned} A \times B &= \{0, 3, 2, 5, 7\} \times \{1, 4, 8, 7, 9, 10\} \\ &= \{0, 3, 2, 5, 7, 12, 8, 20, 70, 45, 28, 24, 16, 40, 56, \\ &\quad 21, 63, 30, 50, 14, 35, 49, 27, 18\}. \end{aligned}$$

T_s is distinctly different from $T_{\cap}$, $T_{\cup}$ and T_o.

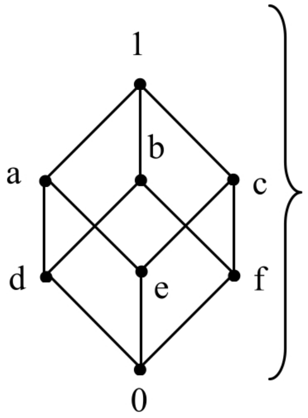
$P_1 = 3Z^+ \cup \{0\} \subseteq Z^+ \cup \{0\}$ be the subring $Z^+ \cup \{0\}$.

If M_1 is the collection of all subset set semiring ideals of S over P_1. We see relative to M_1 we have the new special type of subset set ideal semiring topological spaces relative to P_1 say $T_o^{P_1}$, $T_{\cap}^{P_1}$, $T_{\cup}^{P_1}$ and $T_S^{P_1}$.

We can show all the four are different.

Further each A ∈ M_1, is of infinite order.

Example 4.13: Let S = {Collection of all subsets from the semiring



be the subset semiring of type II over the semiring B.

We see all the four special types of subset topological semiring spaces are of finite order and are different.

Let A = {a, b, c, 0} and B = {d, e, 0, 1} ∈ T_o

A ∪ B = {a, b, c, 0} ∪ {d, e, 0, 1}
= {0, 1, a, d, e, b, c} and

A ∩ B = {a, b, c, 0} ∩ {d, e, 0, 1}
= {0} are in T_o.

T_o is the special new type ordinary subset semiring topological space of S.

Let A, B ∈ $T_\cup$ we see

A ∪ B = {a, b, c, 0} ∪ {d, e, 0, 1}
= {0, a, b, c, e, d, 1}

and A × B = {a, b, c, 0} × {d, e, 0, 1}
= {0, a, b, c, e, d} ∈ $T_\cup$

Clearly $T_\cup$ and T_o are different as topological spaces.

Let A, B $\in T_\cap$ we see

$$\begin{aligned} A + B &= \{a, b, c, 0\} + \{d, e, 0, 1\} \\ &= \{e, d, 0, 1, a, b, c\} \end{aligned}$$

and $$\begin{aligned} A \cap B &= \{a, b, c, 0\} \cap \{d, e, 0, 1\} \\ &= \{0\} \in T_\cap. \end{aligned}$$

We see $T_\cap$ is distinct from T_o and $T_\cup$ as subset topological spaces.

Consider A, B $\in T_s$, we see

$$\begin{aligned} A + B &= \{a, b, c, 0\} + \{d, e, 0, 1\} \\ &= \{0, a, b, e, d, 0, 1, c\} \end{aligned}$$

and $$\begin{aligned} A \times B &= \{a, b, c, 0\} \times \{d, e, 0, 1\} \in T_o \\ &= \{0, a, b, c, e, d\} \in T_s. \end{aligned}$$

T_s is clearly different from all the three spaces T_o, $T_\cup$ and $T_\cap$ as subset topological spaces.

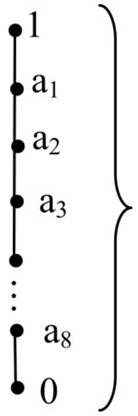
Let us consider $P_1 = \{0, a, d, 1\} \subseteq B$, P_1 is a subsemiring of B.

M_1 = {Collection of all subsets set ideals of S related to the subsemiring P_1}

$$= \{\{0\}, \{0, d\}, \{0, a, d\}, \{0, e\}, \{0, e, a\} \ldots\}$$

Using each subsemiring of B we can get related four special new type subset set ideal topological semiring spaces of type II.

Example 4.14: Let S = {Collection of all subsets from the semiring C_{10} =



be the subset semiring of type II over the semiring C_{10}.

$S = \{\{0\}, \{a_1\}, \ldots, \{a_8\}, \{1\}, \{0, a_1\}, \ldots, \{0, a_8\}, \{1, a_1\}\ \{1, a_8\}, \ldots, \{a_1, a_2\}, \{a_1, a_3\} \ldots, \{a_7, a_8\}, \ldots, \{0, 1, a_1, \ldots, a_8\}\} = T_\cup$ ($T_\cap$ or T_o or T_s).

$A = \{0, 1, a_2, a_4, a_3, a_6\}$ and $B = \{a_5, a_8, a_1, 0\} \in T_o$

We find

$$\begin{aligned} A \cup B &= \{0, 1, a_2, a_4, a_3, a_6\} \cup \{a_5, a_8, a_1, 0\} \\ &= \{0, 1, a_1, a_2, a_3, a_4, a_5, a_6, a_8\} \text{ and} \end{aligned}$$

$$\begin{aligned} A \cap B &= \{0, 1, a_2, a_4, a_3, a_6\} \cap \{a_5, a_8, a_1, 0\} \\ &= \{0\} \in T_o. \end{aligned}$$

For $A, B \in T_\cup$.

$$\begin{aligned} A \cup B &= \{0, 1, a_2, a_4, a_3, a_6\} \cup \{a_5, a_8, a_1, 0\} \\ &= \{0, 1, a_1, a_2, a_3, a_4, a_5, a_6, a_8\} \text{ and} \end{aligned}$$

$$\begin{aligned} A \times B &= \{0, 1, a_2, a_4, a_3, a_6\} \cup \{a_5, a_8, a_1, 0\} \\ &= \{0, a_1, a_5, a_8, a_2, a_3, a_4, a_8\} \in T_\cup. \end{aligned}$$

We see T_o and $T_\cup$ are two different topological new type of subset semiring spaces of type II.

Let A, B $\in T_\cap$,

$$\begin{aligned} A + B &= \{0, 1, a_2, a_4, a_3, a_6\} + \{a_5, a_8, a_1, 0\} \\ &= \{0, 1, a_1, a_5, a_8, a_2, a_3, a_4, a_6\} \text{ and} \end{aligned}$$

$$\begin{aligned} A \cap B &= \{0, 1, a_2, a_4, a_3, a_6\} \cap \{a_5, a_8, a_1, 0\} \\ &= \{0\} \in T_\cap. \end{aligned}$$

Clearly $T_\cap$ is different from $T_\cup$ and T_o as subset topological spaces of type II.

Take A, B $\in T_s$;

$$\begin{aligned} A + B &= \{0, 1, a_2, a_4, a_3, a_6\} + \{a_5, a_8, a_1, 0\} \\ &= \{0, 1, a_1, a_2, a_3, a_4, a_5, a_6, a_8\} \text{ and} \end{aligned}$$

$$\begin{aligned} A \times B &= \{0, 1, a_2, a_4, a_3, a_6\} \times \{a_5, a_8, a_1, 0\} \\ &= \{0, 1, a_1, a_5, a_8, a_2, a_6, a_3, a_4\} \in T_s. \end{aligned}$$

We see T_s is different from T_o, $T_\cup$ and $T_\cap$ as topological spaces.

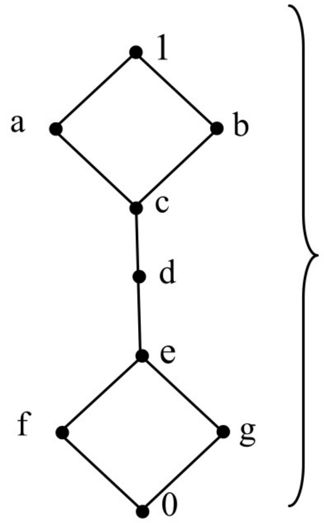
We can by taking subsemiring P_1 of C_{10} we can build

M_1 = {Collection of all set ideal semiring related to the subsemiring P_1 of C_{10}}.

We can find $T_o^{M_1}$, $T_\cup^{M_1}$, $T_\cap^{M_1}$ and $T_S^{M_1}$ and all the four set ideal subset topological semiring spaces of special new type.

We see all the four are different as topological spaces.

Example 4.15: Let S = {Collection of all subsets from the semiring L =



be the subset semiring of type I.

S = {{0}, {a}, {b}, {c}, {d}, …, {0, a, b, c, d, e, f, g}}
= $T_\cup$ ($T_\cap$ or T_o or T_s).

Take A = {0, 1, c, e, g} and B = {a, b, 0, d, f, g} $\in T_\cup$.

We see

$$\begin{aligned} A \cup B &= \{0, 1, c, e, g\} \cup \{a, b, 0, d, f, g\} \\ &= \{0, 1, e, c, g, a, b, d, f\} \text{ and} \end{aligned}$$

$$\begin{aligned} A \times B &= \{0, 1, c, e, g\} \times \{a, b, 0, d, f, g\} \\ &= \{0, a, b, d, f, g, e, c\} \in T_\cup. \end{aligned}$$

Consider A, B $\in T_\cap$.

$$\begin{aligned} A + B &= \{0, 1, c, e, g\} + \{a, b, 0, d, f, g\} \\ &= \{0, 1, e, c, g, a, b, d, f, g\} \text{ and} \end{aligned}$$

$$\begin{aligned} A \cap B &= \{0, 1, c, e, g\} \cap \{a, b, 0, d, f, g\} \\ &= \{0, g\} \text{ are in } T_\cap. \end{aligned}$$

Clearly $T_\cup$ is different from $T_\cap$ as topological of type II.

Consider A, B $\in T_o$.

$$A \cap B = \{0, 1, c, e, g\} \cup \{a, b, 0, d, f, g\}$$
$$= \{0, g\} \text{ and}$$

$$A \cup B = \{0, 1, c, e, g\} \cup \{a, b, 0, d, f, g\}$$
$$= \{0, e, c, 1, g, a, b, f\}.$$

are in T_o.

Further T_o is different from $T_\cup$ and $T_\cap$ as topological of type II.

Now consider A, B $\in T_s$.

We see

$$A + B = \{0, 1, c, e, g\} + \{a, b, 0, d, f, g\}$$
$$= \{0, e, c, 1, g, a, b, f\} \text{ and}$$

$$A \times B = \{0, 1, c, e, g\} \times \{a, b, o, d, f, g\}$$
$$= \{0, a, b, f, g, e, c, g\}$$

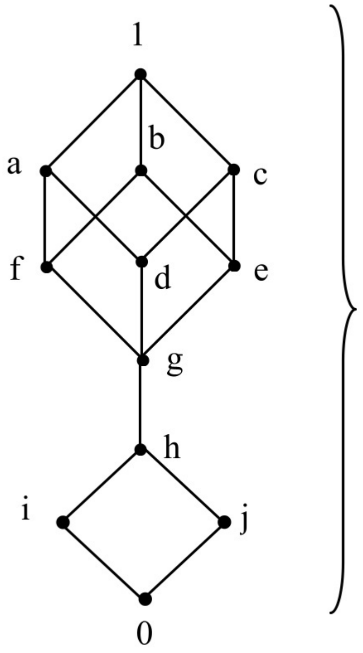
are in T_s and T_s is different from T_o, $T_\cup$ and $T_\cap$ as topological of type II.

Hence we have four different special new type of subset semiring topological spaces of S.

Now if we take $P_1 = \{0, f, e, g\}$ a subsemiring of the semiring and if M_1 = {Collection of all subsets set semiring ideal of S over the semiring P_1}; then $T_o^{P_1}$, $T_\cap^{P_1}$, $T_\cup^{P_1}$ and $T_S^{P_1}$ are special new type of a set semiring ideal topological spaces of M_1.

We can have as many as subset set ideal semiring topological spaces as the number of subsemirings in L of the semiring.

Example 4.16: Let S = {Collection of all subsets of the semiring LS_3 where L =



be the subset semiring; clearly S is a non commutative semiring of LS_3.

We see T_o, $T_\cup$, $T_\cap$ and T_s are the subset special new type of topological spaces of S.

We see if A, B $\in T_o$

say A = $\{ag_1 + g_2 + dg_3, 1 + eg_4, h + ig_5\}$ and
B = $\{bg_2 + g + ig_1, dg_3, h + ig_5\} \in T_o$

We see

$$A \cup B = \{ag_1 + g_2 + dg_3 + 1 + eg_4, h + ig_5\} \cup \{bg_2 + g + ig_1, dg_3, h + ig_5\}$$

$$= \{ag_1 + g_2 + dg_3, 1 + eg_4, h + ig_5, bg_2 + g + ig_1, dg_3\}$$

and

$$\begin{aligned} A \cap B &= \{ag_1 + g_2 + dg_3, 1 + eg_4, h + ig_5\} \cap \{bg_2 + g + ig_1, \\ &\quad dg_3, h + ig_5\} \\ &= \{h + ig_5\} \in T_o. \end{aligned}$$

We see T_o is a commutative new special subset ordinary topological semiring space of S. It is easily verified T_o is always a commutative topological space for $A \cap B = B \cap A$ and $A \cup B = B \cup A$.

Now consider $A, B \in T_\cup$.

$$\begin{aligned} A \cup B &= \{ag_1 + g_2 + dg_3, 1 + eg_4, h + ig_5\} \cup \{bg_2 + g + ig_1, \\ &\quad dg_3, h + ig_5\} \\ &= \{ag_1 + g_2 + dg_3, 1 + eg_4, h + ig_5, bg_2+g + ig_1, dg_3\} \end{aligned}$$

and

$$\begin{aligned} A \times B &= \{ag_1 + g_2 + dg_3, 1 + eg_4, h + ig_5\} \times \{bg_2 + g + ig_1, \\ &\quad dg_3, h + ig_5\} \\ &= \{(ag_1 + g_2 + dg_3) (bg_2 + g + ig_1), (ag_1 + g_2 + dg_3) \\ &\quad dg_3, (ag_1 + g_2 + ig_1) (h + ig_5), (1 + eg_4) (bg_2 + g + \\ &\quad ig_1), (1 + eg_4) dg_3, (1 + eg_4) (h + ig_5), (h + ig_5) \\ &\quad (bg_2 + g + ig1), (h + ig_5) dg_3, (h + ig_5) (h + ig_5)\} \\ &= \{(ag_1 + g_2 + dg_3) (bg_2 + g + ig_1), (dg_4 + dg_5 + d), \\ &\quad (ag_1 + g_2 + ig_1) (h + ig_5), (1 + eg_4) (bg_2 + g + ig_1), \\ &\quad (dg_3 + gg_1), (1 + eg_4) (h + ig_5), (h + ig_5) (bg_2 + g \\ &\quad + ig_1) (hg_3 + ig_2), (h + ig_5) (h + ig_5)\} \qquad \ldots \text{I} \end{aligned}$$

are in $T_\cup$.

But consider

$$\begin{aligned} B \times A &= \{bg_2 + g + ig_1, dg_3, h + ig_5\} \times \{ag_1 + g_2 + dg_3, \\ &\quad 1 + eg_4, h + ig_5\} \\ &= \{(bg_2 + g + ig_1) \times (ag_1 + g_2 + dg_3), dg_3 (ag_1 + g_2 + \\ &\quad dg_3), (h + ig_5) (ag_1 + g_2 + dg_3), (bg_2 + g + ig_1) \end{aligned}$$

$(1 + eg_4)\ dg_3\ (1+ eg_4), (h + ig_5)\ (1 + eg_4),$
$(bg_2 + g + ig_1)\ (h + ig_5), dg_3\ (h + ig_5),$
$(h + ig_5)\ (h + ig_5)\}$

$= \{(bg_2 + g + ig_1)\ (ag_1 + g_2 + dg_3), (d + dg_4 + dg_5), (h + ig_5), (ag_1 + g_2 + dg_3), (bg_2 + g + ig_1)\ (1 + eg_4), (h + ig_5)\ (1 + eg_4)\ (h + ig_5)\ (h + ig_5), (dg_3 + gg_2), (bg_2 + g + ig_1) \times (h + ig_5), (hg_3 + ig_1)\}$ … II

Clearly I and II are distinct. Thus A × B ≠ B × A.

So $T_\cup$ is not a commutative special new type subset topological space semiring space and $T_\cup$ is different from T_o as a topological of type II due to the operations on them.

New consider $T_\cap$; let $A, B \in T_\cap$ we have

$A + B = \{ag_1 + g_2 + dg_3, 1 + eg_4, h + ig_5\} + \{bg_2 + g + ig_1, dg_3, h + ig_5\}$

$= \{ag_1 + g_2 + dg_3 + bg_2 + g + ig_1, 1 + eg_4 + bg_2 + g + ig_1, h + ig_5 + bg_2 + g + ig_1, ag_1 + g_2 + dg_3 + dg_3, 1 + eg_4 + dg_3, h + ig_5 + dg_3, ag_1 + g_2 + dg_3 + h + ig_5, 1+ eg_4 + h + ig_5, h + ig_5 + h + ig_5\}$

$= \{g + dg_3 + g_2 + ag_1, 1 + eg_4 + bg_2 + ig_1, h + ig_5, 1 + eg_4 + ig_5, ag_1 + g_2 + dg_3 + h + ig_5, h + ig_5 + dg_3, 1 + eg_4 + dg_3, ag_1 + g_2 + dg_3, ig_5 + bg_2 + g + ig_1\}$

and

$A \cap B = \{ag_1 + g_2 + dg_3, 1 + eg_4, h + ig_5\} \cap \{bg_2 + g + ig_1, dg_3, h + ig_5\}$

$= \{h + ig_5\} \in T_\cap.$

Thus $T_\cap$ is different from $T_\cup$ and T_o as subset topological space.

Now for A, B $\in T_s$ we find

$$A + B = \{ag_1 + g_2 + dg_3, 1 + eg_4, h + ig_5\} + \{bg_2 + g + ig_1, dg_3, h + ig_5\}$$

$$= \{g + dg_3 + g_2 + ag_1, 1 + eg_4 + bg_2 + ig_1, h + ig_5, 1 + eg_4 + ig_5, ag_1 + g_2 + dg_3 + h + ig_5, h + ig_5 + dg_3, 1 + eg_4 + dg_3, ag_1 + g_2 + dg_3, ig_5 + bg_2 + g + ig_1\}$$

and

$$A \times B = \{ag_1 + g_2 + dg_3, 1 + eg_4, h + ig_5\} \times \{bg_2 + g + ig_1, dg_3, h + ig_5\}$$

$$= \{(ag_1 + g_2 + dg_3) \times (bg_2 + g + ig_1), (ag_1 + g_2 + dg_3) \times dg_3, (ag_1 + g_2 + dg_3) \times (h + ig_5), (1 + eg_4) \times (bg_2 + g + ig_1), (1 + eg_4) \times dg_3, (h + ig_5) \times dg_3, (ag_1 + g_2 + dg_3) \times (h + ig_5), (1 + eg_4) \times (h + ig_5), (h + ig_5) \times (h + ig_5)\} \in T_s.$$

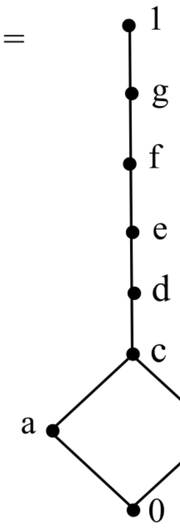
We see T_s is different from $T_\cup$, $T_\cap$ and T_o as subset topological spaces.

Further $A \times B \neq B \times A$ in T_s. We have T_s and $T_\cup$ to be two non commutative topological spaces and $T_\cap$ and T_o are both commutative topological spaces.

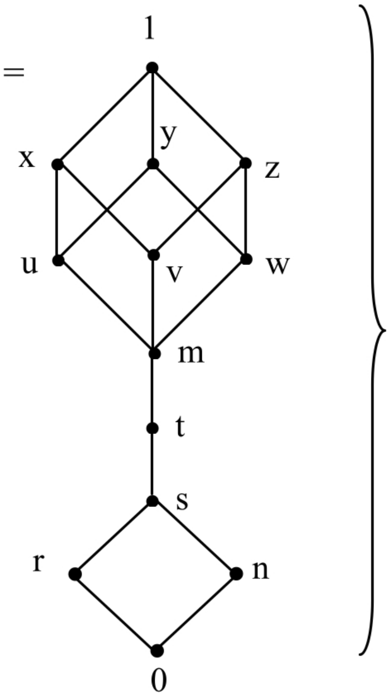
We have seen using this non commutative semirings we have two non commutative topological spaces T_s and $T_\cup$.

Example 4.17: Let S = {Collection of all subsets from the semiring $R = (L_1 \times L_2) D_{2,9}$ where

L_1 =



and L_2 =



be the subset semiring of type II.

We have all the four topological spaces $T_{\cup}$, $T_{\cap}$, T_o and T_s and all of the four spaces are distinct but $T_{\cup}$ and T_s are non commutative where as $T_{\cap}$ and T_o are commutative spaces. All the four spaces are of finite order.

Example 4.18: Let S = {Collection of all subsets from the semiring $\langle Z^+ \cup \{0\} \cup I\rangle S_4$} be the subset semiring. $T_{\cup}$, T_o, $T_{\cap}$ and T_s are the four topological spaces of S.

T_o and $T_{\cap}$ are commutative where as $T_{\cup}$ and T_s are non commutative.

Now having seen examples of topological spaces we now define new special subset set ideal topological semiring spaces over the subsemiring of all the four types.

Example 4.19: Let S = {Collection of all subsets from the semiring R = $\langle Q^+ \cup I\rangle \cup \{0\}$} be the subset semiring. All the four topological spaces T_o, $T_{\cup}$, $T_{\cap}$ and T_s are of infinite order and all of them are commutative.

We see $(Z^+ \cup \{0\}) = P_1$ is a subsemiring of $\langle Q^+ \cup I\rangle \cup \{0\}$.

M_1 = {Collection of all set ideals of semiring relative to the subsemiring $(Z^+ \cup \{0\})) = P_1$}.

We see $T_o^{P_1}$, $T_\cap^{P_1}$, $T_\cup^{P_1}$ and $T_S^{P_1}$ are special new type of subset set semiring ideal topological spaces relative to P_1. All of them are of infinite order.

If we take $P_2 = \langle 2Z^+ \cup \{0\}\rangle \subseteq \langle Q^+ \cup I\rangle \cup \{0\}$ we get another collection of topological spaces.

Infact we have infinite number of such topological spaces all of which are of infinite order.

Example 4.20: Let S = {Collection of all subsets from the semiring LS(5) where L = Boolean algebra of order 16} be the subset semiring. T_o, $T_\cup$, $T_\cap$ and T_s are all the four distinct special subset new topological semiring spaces of type II of S.

All of them are of finite order but only $T_\cup$ and T_s are non commutative.

Now having seen examples of these spaces we just describe the notion of strong special new subset topological semiring spaces over subset semiring.

Example 4.21: Let
S = {Collection of all subsets from the ring Z_6} be the subset semiring of type I of the ring Z_6.

Take P_1 = {All subsets from the semiring $\{0, 2, 4\} \subseteq Z_6$} = {{0}, {2}, {4}, {0, 2}, [0, 4}, {2, 4}, {0, 2, 4}} ⊆ S.

Clearly P_1 is again a subset semiring.

Now let M_1 = {Collection of all subset set ideals of S over the subset semiring P_1 of S}

$$= \{\{0\}, \{1, 2, 4, 0\}, \{2, 4, 0\}, \{0, 2, 4, 3\}, \{3, 0\}, \{0, 3, 1, 2, 4\}, \{0, 5, 4, 2\}, \{1, 5, 2, 4, 0\}, \{0, 5, 1, 2, 3, 4\}, \{0, 5, 2, 4, 3\}\}.$$

On M_1 we can have the four distinct strong special new subset semiring set ideal topological spaces $T_S^{P_1}$, $T_o^{P_1}$, $T_\cap^{P_1}$ and $T_\cup^{P_1}$.

Let A = {0, 5, 2, 4} and B = {0, 2, 3, 4} $\in T_o^{P_1}$.

$$\begin{aligned} A \cup B &= \{0, 5, 2, 4\} \cup \{0, 2, 3, 4\} \\ &= \{0, 2, 4, 3, 5\} \text{ and} \end{aligned}$$

$$\begin{aligned} A \cap B &= \{0, 5, 2, 4\} \cap \{0, 2, 3, 4\} \\ &= \{0, 2, 4\} \in T_o^{P_1}. \end{aligned}$$

We see $T_o^{P_1}$ is a strong special subset set ideal semiring topological spaces associated with the subset semiring P_1.

Take A, B $\in T_\cup^{P_1}$;

$$\begin{aligned} A \cup B &= \{0, 5, 2, 4\} \cup \{0, 2, 3, 4\} \\ &= \{0, 2, 4, 3, 5\} \text{ and} \end{aligned}$$

$$\begin{aligned} A \times B &= \{0, 5, 2, 4\} \times \{0, 2, 3, 4\} \\ &= \{0, 4, 2, 3\} \text{ are in } T_\cup^{P_1}. \end{aligned}$$

Clearly $T_\cup^{P_1}$ is different from $T_o^{P_1}$ as subset set ideal semiring topological space.

Now consider A, B $\in T_\cap^{P_1}$;

$$\begin{aligned} A + B &= \{0, 5, 2, 4\} + \{0, 2, 3, 4\} \\ &= \{0, 1, 4, 3, 2, 5\} \text{ and} \end{aligned}$$

$$\begin{aligned} A \cap B &= \{0, 5, 2, 4\} \cap \{0, 2, 3, 4\} \\ &= \{0, 2, 4\} \in T_{\cap}^{P_1}. \end{aligned}$$

We see $T_{\cap}^{P_1}$ is different from $T_{o}^{P_1}$ and $T_{\cup}^{P_1}$ as subset set ideal semiring topological spaces.

Consider A, B $\in T_{S}^{P_1}$;

$$\begin{aligned} A + B &= \{0, 5, 2, 4\} + \{0, 2, 3, 4\} \\ &= \{0, 1, 2, 3, 4, 5\} \text{ and} \end{aligned}$$

$$\begin{aligned} A \times B &= \{0, 5, 2, 4\} \times \{0, 2, 3, 4\} \\ &= \{0, 2, 4, 3\} \text{ are in } T_{S}^{P_1}. \end{aligned}$$

Clearly $T_{S}^{P_1}$ is also different from $T_{o}^{P_1}$, $T_{\cup}^{P_1}$ and $T_{\cap}^{P_1}$ as subset set ideal semiring topological spaces.

All the strong set ideal subset special new semiring topological spaces over P_1 are distinct and of finite order.

With $P_2 = \{0, 3\} \subseteq Z_6$ we can built M_2 = {Collection of all subsets from the subring $P_2 = \{0, 3\} \subseteq Z_6\} = \{\{0\}, \{3\}, \{0, 3\}\}$. Associated with M_2 we have four strong subset set ideal semiring topological spaces of P_2 given by $T_{o}^{P_2}$, $T_{\cup}^{P_2}$, $T_{\cap}^{P_2}$ and $T_{S}^{P_2}$.

Example 4.22: Let
S = {Collection of all subsets from the ring $Z_7(g)$ where $g^2 = 0$} be the subset semiring of the ring $Z_7(g)$. We see $P_1 = Z_7 \subseteq Z_7(g)$ is a subring of $Z_7(g)$. Let M_1 = {Collection of all set subsets ideal semirings of S over the subring Z_7}

$$\begin{aligned} = \; & \{\{0\}, \{0, 1, 2, 3, \ldots, 6\}, \{0, g, 2g, \ldots, 6g\}, \\ & \{1+g, 2+2g, \ldots, 6+6g, 0\}, \ldots\}. \end{aligned}$$

Using M_1 we can have subset set ideal special strong semiring topological spaces $T_o^{P_1}$, $T_{\cup}^{P_1}$, $T_{\cap}^{P_1}$ and $T_S^{P_1}$.

Let A = {0, 1+2g, 2+4g, 6+g, 5+2g, 3+4g, 4+g, 5+3g, 6+5g, 3+6g, 4+3g, 2+5g, 1+6g} and B = {0, 1, 2, 3, 4, 5, 6} $\in T_o^{P_1}$.

$$A \cup B = \{0, 1+2g, 2+4g, 6+g, 5+2g, 3+4g, 4+g, 5+3g, 6+5g, 3+6g, 2+5g, 4+3g, 1+6g\} \cup \{0, 1, 2, 3, 4, 5, 6\}$$

$$= \{0, 1, 2, 3, 4, 5, 6, 1+2g, 2+4g, 6+g, 5+2g, 3+4g, 4+g, 5+3g, 6+5g, 3+6g, 2+5g, 4+3g, 1+6g\} \text{ and}$$

$$A \cap B = \{0, 1+2g, 2+4g, 6+g, 5+2g, 3+4g, 4+g, 5+3g, 6+5g, 3+6g, 2+5g, 4+3g, 1+6g\} \cap \{0, 1, 2, 3, 4, 5, 6\}$$

$$= \{0\} \in T_o^{P_1}.$$

Consider A, B $\in T_{\cup}^{P_1}$.

$$A \cup B = \{0, 1+2g, 2+4g, 6+g, 5+2g, 3+4g, 4+g, 5+3g, 6+5g, 3+6g, 2+5g, 4+3g, 1+6g\} \cup \{0, 1, 2, 3, 4, 5, 6\}$$

$$= \{0, 1, 2, 3, 4, 5, 6, 1+2g, 2+4g, 6+g, 5+2g, 3+4g, 4+g, 5+3g, 6+5g, 3+6g, 2+5g, 4+3g, 1+6g\} \text{ and}$$

$$A \times B = \{0, 1+2g, 2+4g, 6+g, 5+2g, 3+4g, 4+g, 5+3g, 6+5g, 3+6g, 2+5g, 4+3g, 1+6g\} \times \{0, 1, 2, 3, 4, 5, 6\}$$

$$= \{0, 1+2g, 2+4g, 6+g, 5+2g, 3+4g, 4+g, 3g+5, 6+5g, 3+6g, 2+5g, 4+3g, 1+6g\}$$

$= A \in T_{\cup}^{P_1}$.

$T_{\cup}^{P_1}$ is different from $T_{o}^{P_1}$ as topological spaces.

Let A, B $\in T_{\cap}^{P_1}$;

$A + B = \{0, 1+2g, 2+4g, 6 + g, 5 + 2g, 3 + 4g, 4 + g, 5 + 3g, 6 + 5g, 3 + 6g, 2 + 5g, 4 + 3g, 1 + 6g\} + \{0, 1, 2, 3, 4, 5, 6\}$

$= \{0, 1, 2, 3, 4, 5, 6, g, 2g, 3g, 4g, 5g, 6g, 1+2g, 2 + 4g, 6 + g, 5 + 2g, 3 + 4g, 4 + g, 5 +3g, 6 + 5g, 3 + 6g, 4 + 3g, 2 + 5g, 1 + 6gh, 2 + 4g, 2 + 2g, 3 + 2g, 4 + 2g, 6 + 2g, 4 + 4g, 3 + 3g, 5 + 5g, 6 + 6g\} \ldots\}$ and

$A \cap B = \{0, 1+2g, 2+4g, 6 + g, 5 + 2g, 3 + 4g, 4 + g, 5 + 3g, 6 + 5g, 3 + 6g, 2 + 5g, 4 + 3g, 1 + 6g\} \cap \{0, 1, 2, 3, 4, 5, 6\}$
$= \{0\} \in T_{\cap}^{P_1}$.

$T_{\cup}^{P_1}$ and $T_{o}^{P_1}$ are different as topological spaces from $T_{\cap}^{P_1}$.

Consider A, B $\in T_{S}^{P}$ we can show all the four spaces are distinct.

Example 4.23: Let S = {Collection of all subsets from the Z_{12}} be the subset semiring of type I.

$P_1 = \{0, 3, 6, 9\} \subseteq Z_{12}$ be the subring. $W_1 = \{\{0\}, \{3\}, \{6\}, \{9\}, \{0, 3\}, \{0, 6\}, \{0, 9\}, \{3, 6\}, \{3, 9\}, \{6, 9\}, \{0, 3, 6\}, \{0, 3, 9\}, \{0, 6, 9\}, \{3, 6, 9\}, \{0, 3, 6, 9\}\}$ be the subset semiring.

Let M_1 = Collection of all subset set semiring ideals of the subset semiring over the subset subsemiring W_1.

We can find the four strong special new type subset set semiring ideals of S relative to the subset subsemiring W_1.

We see $T_o^{W_1}$, $T_\cap^{W_1}$, $T_\cup^{W_1}$, and $T_S^{W_1}$ are 4 different strong special subset set semiring ideal topological spaces of S relative to W_1.

Example 4.24: Let
S = {Collection of all subsets from the ring Z_6S_3} be the subset semiring of type I.

P_1 = {Collection of all subsets from the subring $\{0, 3\}S_3$} be the subset subsemiring of type I.

P_2 = {Collection of all subsets from the subring $\{0, 2, 4\}S_3$} be the subset subsemiring of type I.

P_3 = {Collection of all subsets from the subring Z_6A_3} be the subset subsemiring of type I.

P_4 = {Collection of all subsets from the subring $\{0,3\}A_3$} be the subset subsemiring of type I.

P_5 = {Collection of all subsets from the subring $\{0,2,4\}A_3$} be the subset subsemiring of type I.

P_6 = {Collection of all subsets from the subring Z_6W_1 where

$$W_1 = \left\{\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}\right\} \subseteq S_3 \text{ is a subgroup of } S_3\}$$

be the subset subsemiring of type I.

P_7 = {Collection of all subsets from the subring Z_6W_2 where

$$W_2 = \left\{\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}\right\} \subseteq S_3\}$$

be the subset subsemiring of type I.

P_8 = {Collection of all subsets from the subring Z_6W_3 where

$$W_3 = \left\{\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}\right\}\}$$

be the subset subsemiring of type I.

P_9 = {Collection of all subsets from the subring {0, 3}W_1} be the subset subsemiring of type I.

P_{10} = {Collection of all subsets from the subring {0, 3}W_2} be the subset subsemiring of type I.

P_{11} = {Collection of all subsets from the subring {0, 3}W_3} be the subset subsemiring of type I.

Similarly P_{12}, P_{13}, and P_{14} are subset subsemirings of the subring {0, 2, 4}W_i; i = 1, 2, 3.

Thus we have nearly 14 such subset subsemirings of S.

Related with each of them we have a special subset set ideal semiring topological subspaces of S of the four types as well as the strong special new subset set ideal semiring topological subspaces over P_i of the four types.

Associated with each of these topological spaces we have a tree.

Study in this direction is not only innovative and interesting but has lots of bearing in applications.

Example 4.25: Let
S = {Collection of all subsets from the ring $R(S(3) \times D_{2,7} \times A_4)$} be the subset semiring of type I.

We have several special new subset set semiring ideal topological spaces of S over the subrings of S as well as strong special new subset set semiring ideal topological spaces of S over the subset subsemirings of S.

We can have infinite number of trees of infinite cardinality associated with these spaces.

Now we proceed onto define and describe the notion of subset semiring topological spaces of type II semirings.

Example 4.26: Let
S = {Collection of all subsets from the semiring $Z^+ \cup \{0\}$} be the subset semiring of type II.

We have infinite number of subset subsemiring of S and associated with each one of them we have an infinite number of subset special set ideal semiring new topological semiring spaces of S. We can associate infinite trees with these spaces and their subspaces.

Example 4.27: Let S = {Collection of all subsets from the semiring $\langle Q^+ \cup I \cup \{0\}\rangle$} be the subset semiring of type II. S has infinite number of subset subsemirings.

We also have T_o, $T_\cup$, $T_\cap$ and T_s the four special subset topological semiring spaces of S.

Let $T_o = \{S' = S \cup \{\phi\}, \cup, \cap\}$,
$T_\cup = \{S, \cup, \times\}$,
$T_\cap = \{S', \cap, +\}$ and
$T_s = \{S, +, \times\}$ be the four special subset topological semiring spaces of S.

Take A = {3 + I, 8I, 0, 4+2I, 7I+1} and B = {I, 1, 9+I} $\in$ T_o.

$A \cup B = \{3+I, 8I, 0, 4+2I, 7I+1\} \cup \{I, 1, 9 + I\}$

$= \{3 + 2I, 9I, I, 4 + 3I, 8I + 1, 4 + I, 1+ 8I, 1, 5 + 2I, 2 + 7I, 12 + 2I, 9 + 9I, 9 + I, 13 + 3I, 8I + 10\}$

and

$A \cap B = \{3+I, 8I, 0, 4+2I, 7I+1\} \cap \{I, 1, 9 + I\}$
$= \{\phi\} \in T_o$.

This is the way operations are performed on T_o.

Consider A, B $\in T_s = \{S, \cup, \times\}$.

Now

$A + B = \{3 + I, 8I, 0, 4 + 2I, 7I + 1\} + \{I, 1, 9 + I\}$

$= \{3 + 2I, 9I, I, 4 + 3I, 8I + 1, 4 + I, 1 + 8I, 1, 5 + 2I, 2 + 7I, 12 + 2I, 9 + 9I, 9 + I, 13 + 3I, 8I + 10\}$

and

$A \times B = \{3 + I, 8I, 0, 4 + 2I, 7I + 1\} \times \{I, 1 + 9 + I\}$

$= \{3 + I, 8I, 0, 4 + 2I, 7I + 1, 4I, 8I, 6I, 27 + 13I, 80I, 36 + 24I, 9 + 71I\}$ are in T_s.

Clearly T_o and T_s are two distinct special subset topological spaces of the semiring.

Let A, B $\in T_\cap = \{S', +, \cap\}$.

Now

$A + B = \{3 + I, 8I, 0, 4 + 2I, 7I + 1\} + \{I, 1, 9 + I\}$

$= \{3 + 2I, 9I, I, 4+3I, 8I + 1, 4 + I, 1 + 8I, 1, 5+2I, 2+7I, 12 + 2I, 9+9I, 9+I, 13+3I, 8I + 10\}$ and

$A \cap B = \{3 + I, 8I, 0, 4 + 2I, 7I + 1\} \cap \{I, 1, 9 + I\}$
$= \{\phi\}$ are in $T_\cap$.

Let A, B $\in T_{\cup} = \{S, \cup, \times\}$.

$A \cup B = \{3 + I, 8I, 0, 4 + 2I, 7I + 1\} \cup \{I, 1, 9 + I\}$

$= \{3+I, 8I, 0, 4+ 2I.7I + 1, I, 1, 9+I\}$ and

$A \times B = \{3 + I, 8I, 0, 4 + 2I, 7I + 1\} \times \{I, 1, 9 + I\}$

$= \{4I, 8I, 0, 6I, 8I, 3 + I, 8I, 0, 4 + 2I, 27 + 13I, 73I, 36 + 24I, 9 + 77I\} \in T_{\cup}$.

We see all the four topological spaces T_o, $T_{\cup}$, $T_{\cap}$ and T_s are distinct.

Example 4.28: Let S = {Collection of all subsets from the semiring $(Z^+ \cup \{0\})$ (g_1, g_2, g_3) where $g_1^2 = 0$, $g_2^2 = g_2$ and $g_3^2 = g_3$, with $g_ig_j = g_jg_i = 0$ if $i \neq j$, $1 \leq i, j \leq 3$} be the subset semiring of type II.

Let T_o, $T_{\cup}$, $T_{\cap}$ and T_s be the four new special subset semiring topological spaces of S.

Let $A = \{0, 2 + g_2 + g_1, 4g_1 + 5g_3\}$ and $B = \{g_2, 4g_3, 5g_1, 3+2g_1 + g_3\} \in T_o$.

$\{S' = S \cup \{\phi\}, \cup, \cap\}$. We now find

$A \cup B = \{0, 2 + g_2 + g_1, 4g_1 + 5g_3\} \cup \{g_2, 4g_3, 5g_1, 3+2g_1 + g_3\}$

$= \{0, 2 + g_1 + g_2, 4g_1 + 5g_3, g_2\ 4g_3, 5g_1, 3 + 2g_1 + g_3\}$ and

$A \cap B = \{0, 2 + g_2 + g_1, 4g_1 + 5g_3\} \cap \{g_2, 4g_3, 5g_1, 3+2g_1 + g_3\}$

$= \phi \in T_o$.

Let A, B $\in T_{\cup} = \{S, \cup, \times\}$ we find

$A \cup B = \{0, 2 + g_2 + g_1, 4g_1 + 5g_3\} \cup \{g_2, 4g_3, 5g_1, 3+2g_1 + g_3\}$

$= \{0, 2 + g_1 + g_2, 4g1 + 5g_3, g_2, 4g_3, 5g_1, 3+2g_1\}$

and

$A \times B = \{0, 2 + g_2 + g_1, 4g_1 + 5g_3\} \cup \{g_2, 4g_3, 5g_1, 3+2g_1 + g_3\}$

$= \{0, 3g_2, 8g_3, 20g_3, 10g_1, 6 + 2g_1 + 2g_3\} \in T_{\cup}$.

We see T_o and $T_{\cup}$ are different types of topological spaces of the subset semiring S.

Let A, B $\in T_{\cap} = \{S' = S \cup \{\phi\}, +, \cap\}$ we find
$A + B = \{0, 2 + g_2 + g_1, 4g_1 + 5g_3\} + \{g_2, 4g_3, 5g_1, 3+2g_1 + g_3\}$

$= \{g_2, 4g_3, 5g_1, 3 + 2g_1 + g_3, 2 + g_1 + 2g_2, 2 + g_1 + g_2 + 4g_3, 6g_1 + 2 + g_2, 5 + 3g_1 + g_2 + g_3, 4g_1 + g_2 + 5g_3, 9g_3 + 4g_1, 9g_1 + 5g_3, 3 + 6g_1 + 6g_3\}$ and

$A \cap B = \{0, 2 + g_2 + g_1, 4g_1 + 5g_3\} \cap \{g_2, 4g_3, 5g_1, 3+2g_1 + g_3\}$

$= \phi$ are in $T_{\cap}$.

We see $T_{\cap}$ is distinct from T_o and $T_{\cup}$ as subset topological spaces.

Now let A, B $\in T_s = \{S, +, \times\}$

A + B

$= \{0, 2 + g_2 + g_1, 4g_1 + 5g_3\} + \{g_2, 4g_3, 5g_1, 3+2g_1 + g_3\}$

$= \{g_2, 4g_3, 5g_1, 3 + 2g_1 + g_3, 2 + g_1 + 2g_2, 2 + g_1 + g_2 + 4g_3, 6g_1 + 2 + g_2, 5 + 3g_1 + g_2 + g_3, 4g_1 + g_2 + 5g_3, 9g_3 + 4g_1, 9g_1 + 5g_3, 3 + 5g_1 + 6g_3\}$

and

$A \times B$

$= \{0, 2 + g_2 + g_1, 4g_1 + 5g_3\} \times \{g_2, 4g_3, 5g_1, 3+2g_1 + g_3\}$

$= \{0, 3g_2, 9g_3, 20g_3, 10g_1, 6 + 7g_1 + 3g_2 + 2g_3, 12g_1 + 20g_3\}$
are in T_s.

We see T_s is distinctly different from $T_\cup$, $T_\cap$ and T_o as topological subset spaces.

Thus T_s, T_o, $T_\cup$ and $T_\cap$ are four distinct special new subset semiring topological spaces of S.

Further we have subspaces of these four topological spaces.

We also have the special set ideal semiring subset new type topological semiring spaces related to subset subsemirings.

For we have infinite number of subset subsemiring; $T_o^n = \{S^n \cup \{\phi\}; \cup, \cap\}$ where S^n = {all subsets of the subsemiring $nZ^+ \cup \{0\}$} as n varies in $Z^+ \setminus \{1\}$.

Thus we have infinite number of special new subset set semiring ideal topological semiring spaces over the subset subsemirings S^n of the four types T_o^n, $T_\cup^n$, $T_\cap^n$ and T_S^n.

Apart from this also we have infinite collection of spaces associated with the subset subsemirings

$S^n(g_1)$ = {Collection of all subsets from the subsemiring $(nZ^+ \cup \{0\})(g_1)$); $n \in Z^+ \setminus \{1\}$},

$S^n(g_2)$= {Collection of all subsets from the subsemiring $(nZ^+ \cup \{0\})(g_2)$)},

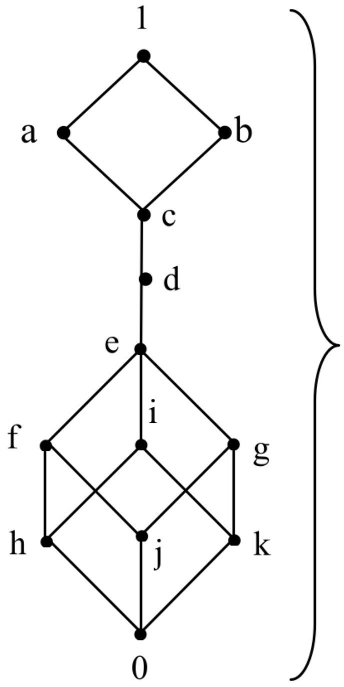
$S^n(g_3)$ = {Collection of all subsets from the subsemiring $(nZ^+ \cup \{0\})\ (g_3)$; $n \in Z^+ \setminus \{1\}$} be subset subsemirings of S.

Likewise $S^n(g_1, g_2)$, $S^n(g_1, g_3)$, $S^n(g_2, g_3)$ and $S^n(g_1, g_2, g_3)$ give way to infinite collection of all subset subsemirings.

Associated with these subset subsemirings we have an infinite collection of the four types of new type of special subset set ideal topological semiring spaces of $T_\cup$, $T_\cap$, T_o and T_s.

Now related with these subset set semiring ideal topological semiring subspaces of the topological spaces we have trees associated with them.

Example 4.29: Let S = {Collection of all subsets from the semiring L where

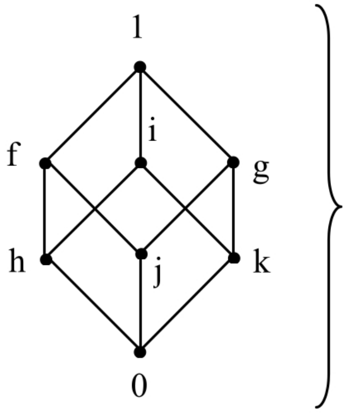


be the subset semiring.

We have T_o, $T_\cup$, $T_\cap$ and T_s to be the four topological spaces.

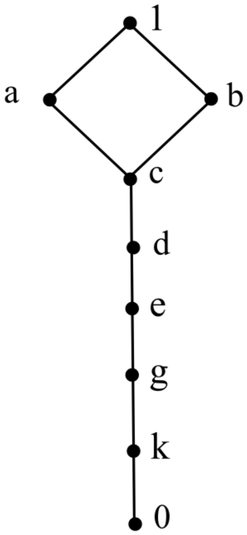
We have also subspaces but only finite in number. Further all the spaces T_o, $T_\cup$, $T_\cap$ and T_s have finite trees associated with them.

Take S_1 = {Collection of all subsets from the sublattice P_1 =



$\subseteq$ S is a subset subsemiring. $T_\cup^{P_1}$, $T_\cap^{P_1}$, $T_o^{P_1}$ and $T_S^{P_1}$ are all special new type subset set semiring ideal topological subspaces of the space $T_\cup$, $T_\cap$, T_o and T_s respectively.

Consider S_2 = {Collection of all subsets from the sublattice (subsemiring) P_2 =



be the subset subsemiring of S.

$T_o^{P_2}$, $T_\cup^{P_2}$, $T_\cap^{P_2}$ and $T_S^{P_2}$ are the subset special new set semiring ideal topological semiring subspaces of the subset subsemiring space S_2 of T_o, $T_\cup$, $T_\cap$ and T_s respectively.

However we have only a finite number of such spaces as S has only a finite number of elements in it.

Take A = {a, b, f, h, 0} and
B = {1, d, k, g, j} $\in T_o = \{S', \cup, \cap\}$.

We find

$$A \cup B = \{a, b, f, h, 0\} \cup \{l, d, k, g, j\}$$
$$= \{a, b, f, h, 0, l, d, k, g, j\} \text{ and}$$

$$A \cap B = \{a, b, f, h, 0\} \cup \{l, d, k, g, j\}$$
$$= \phi \in T_o.$$

Now let A, B $\in T_\cup = \{S, \cup, \times\}$.

$$A \cup B = \{a, b, f, h, 0\} \cup \{l, d, k, g, j\}$$
$$= \{a, b, f, h, 0, l, d, k, g, j\} \text{ and}$$

$$A \times B = \{a, b, f, h, 0\} \times \{l, d, k, g, j\}$$
$$= \{a, b, f, h, 0, d, k, g, j\} \text{ is in } T_\cup.$$

We see T_o and $T_\cup$ are two different topological spaces.

Now consider A, B $\in T_s$.

$$A + B = \{a, b, f, h, 0\} + \{l, d, k, g, j\}$$
$$= \{1, a, b, d, k, e, g, j, f\} \text{ and}$$

$$A \times B = A \cup B = \{a, b, f, h, 0\} \times \{l, d, k, g, j\}$$
$$= \{a, b, f, h, 0, d, k, g, j\} \in T_s.$$

We see T_s is different from T_o and $T_\cup$.

Let A, B $\in T_\cap = \{S' = S \cup \{\phi\}, +, \cap\}$

$$\begin{aligned} A + B &= \{a, b, f, h, 0\} + \{l, d, k, g, j\} \\ &= \{1, a, b, d, i, k, e, g, j, f\} \text{ and} \end{aligned}$$

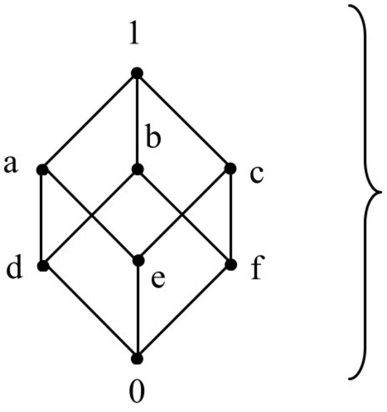
$$\begin{aligned} A \cap B = A \cup B &= \{a, b, f, h, 0\} \cap \{l, d, k, g, j\} \\ &= \phi \text{ are in } T_\cap. \end{aligned}$$

We see $T_\cap$ is different from $T_\cup$, T_o and T_s as topological spaces.

We see in all the four spaces A × B = B × A, hence all the spaces are commutative.

Example 4.30: Let
S = {Collection of all subsets from the semiring LS_3 where L is lattice which is as follows:

L =



be the subset semiring of finite order.

Clearly S is non commutative.

$$\text{For if } A = \left\{\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} + a\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} + d\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix},\right.$$

$$\left. e\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}, c\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}\right\} \text{ and}$$

$$B = \left\{\begin{pmatrix}1&2&3\\1&2&3\end{pmatrix} + d\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}, f\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix},\right.$$

$$\left. c\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}\right\} \text{ are in S we find}$$

$$A + B = \left\{\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix} + a\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix} + d\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix},\right.$$

$$\left. e\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}, c\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}\right\} + \left\{\begin{pmatrix}1&2&3\\1&2&3\end{pmatrix} +\right.$$

$$\left. d\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}, f\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}, c\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}\right\}$$

$$= \left\{\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix} + \begin{pmatrix}1&2&3\\1&2&3\end{pmatrix} + a\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix} +\right.$$

$$d\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}, e\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix} + \begin{pmatrix}1&2&3\\1&2&3\end{pmatrix} + d\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix},$$

$$d\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix} + c\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix} + \begin{pmatrix}1&2&3\\1&2&3\end{pmatrix}, \begin{pmatrix}1&2&3\\2&3&1\end{pmatrix} +$$

$$a\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix} + d\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}, e\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix} +$$

$$d\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}, c\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix} + d\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix},$$

$$c\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}, c\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}+f\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}, \begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}$$

$$+a\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}+d\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}+c\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix},$$

$$\left. e\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}+c\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}\right\}.$$

and

$$A \times B = \left\{\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}+a\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}+d\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix},\right.$$

$$\left. e\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}, c\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}\right\} \times \left\{\begin{pmatrix}1&2&3\\1&2&3\end{pmatrix}+\right.$$

$$\left. d\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}, f\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}, c\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}\right\}$$

$$= \left\{\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}+a\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}+d\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}\right.$$

$$+d\begin{pmatrix}1&2&3\\1&2&3\end{pmatrix}, e\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}, c\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}+$$

$$d\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}, 0, e\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix},$$

$$f\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}, f\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}\Bigg\}.$$

Consider

$$B \times A = \left\{\begin{pmatrix}1&2&3\\1&2&3\end{pmatrix} + d\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}, f\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix},\right.$$

$$\left. c\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}\right\} \times \left\{\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix} + a\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix} + \right.$$

$$\left. d\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}, e\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}, c\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}\right\}$$

$$= \left\{\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix} + a\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix} + d\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix},\right.$$

$$e\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}, c\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}, d\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix},$$

$$\left. f\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}, f\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix} \text{and so on}\right\} \in S.$$

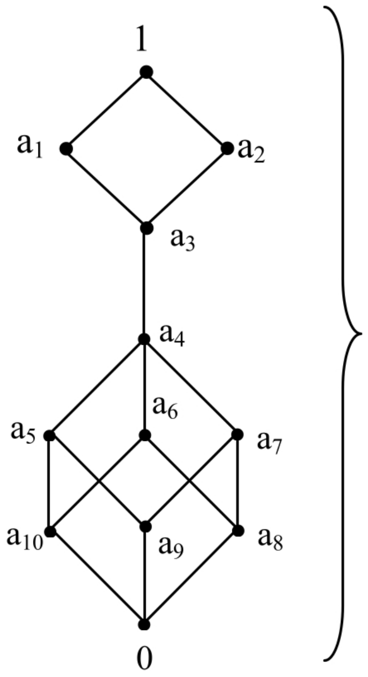
$A \times B \neq B \times A$ in S.

Thus S is a subset semiring which is non commutative so the spaces $T_{\cup}$ and T_s are non commutative.

We can get only finite number of subset subsemirings and their associated subset special new set semiring ideal topological semiring spaces of the subset subsemirings.

Example 4.31: Let S = {Collection of all subsets form the semiring $(L_1 \times L_2)D_{2,7}$ where L_1 is a Boolean algebra of order 16 and L_2 is a chain lattice C_{12}} be the subset semiring of finite order which is non commutative. S has subset special new topological subspaces of 4 types of which $T_\cup$ and T_s are non commutative.

Example 4.32: Let S = {Collection of all subsets from the semiring LS(3) where L =



be the subset semiring of finite order.

We have only finite number of subset special new type of topological subset semirings of which T_s and $T_\cup$ are non commutative spaces of S.

THEOREM 4.1: *Let S be a collection of all subsets of ring (semiring R_1). S be the subset semiring of type I (or type II). S is non commutative if and only if the ring R (or semiring R_1) is non commutative.*

Proof is direct and hence left as an exercise to the reader.

Corollary 4.1: The new special subset semiring topological spaces $T_\cup$ and T_s is non commutative if and only if S is non commutative.

The proof of this corollary is also direct and hence left for the reader to prove.

Example 4.33: Let S = {Collection of all subsets of the semiring V = $(L_1 \times L_2)$ $(D_{2,8} \times A_6)$} be the subset semiring which is of finite order and non commutative.

Take $P_1 = (L_1 \times \{0\}) (\{1\} \times A_6) \subseteq V = (L_1 \times L_2) (D_{2,8} \times A_6)$ to be a subsemiring of V.

W_1 = {Collection of all subsets from subsemiring P_1} $\subseteq$ S is a subset subsemiring. $T_o^{W_1}$, $T_\cup^{W_1}$, $T_\cap^{W_1}$ and $T_s^{W_1}$ are the special new strong subset set semiring ideal topological spaces over the subset subsemiring W_1.

$T_s^{W_1}$ and $T_\cup^{W_1}$ are non commutative but of finite order.

W_1 is also associated with special new subset set ideal semiring topological spaces over the subsemiring P_1 of finite order.

They are $T_S^{P_1}$, $T_o^{P_1}$, $T_\cup^{P_1}$ and $T_\cap^{P_1}$. Compare and study $T_\cup^{P_1}$ with $T_\cup^{W_1}$ and so on.

Example 4.34: Let S = {Collection of all subsets from the semiring W = $(Z^+ \cup \{0\})$ $(S_4 \times D_{2,5})$} be the subset semiring of type II.

Take $P_t = \{(tZ^+ \cup \{0\}) (S_4 \times D_{25})\} \subseteq S$; $t \in Z^+ \setminus \{1\}$ be the collection of all subsemirings of W.

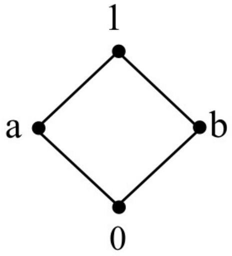
Let V_t = {Collection of all subsets of the subsemiring P_t}; $t \in Z^+ \setminus \{1\}$ be the subset subsemiring of S.

$T_o^{P_t}$, $T_s^{P_t}$, $T_\cup^{P_t}$ and $T_\cap^{P_t}$ are the special subset new type of set ideal semiring topological spaces of S over the subsemiring P_t; $t \in Z^+ \setminus \{1\}$.

$T_o^{W_1}$, $T_s^{W_1}$, $T_\cup^{W_1}$ and $T_\cap^{W_1}$ are strong special subset new type set ideal semiring topological space of S over the subset subsemiring W_1 of S.

We see we have an infinite collection of topological spaces all of infinite order.

Example 4.35: Let S = {Collection of all subsets from the semiring L} be the subset semiring of type II where L =



S = {{0}, {1}, {a}, {b}, {0, 1}, {0, a}, {0, b}, {a, b}, {0, 1, a}, {0, 1, b}, {0, a, b}, {1, a, b}, {1, a, b, 0}}.

T_s, T_o, $T_\cup$ and $T_\cap$ are the four different topological spaces of S.

Let A = {0, 1, b} and B = {a} $\in T_s$.

$$\begin{aligned} A + B &= \{0, 1, b\} + \{a\} \\ &= \{a, 1\} \text{ and} \end{aligned}$$

$$\begin{aligned} A \times B &= \{0, 1, b\} + \{a\} \\ &= \{a, 0\} \in T_s. \end{aligned}$$

For A, B $\in T_\cup = \{S, \cup, \times\}$

$A \cup B = \{0, 1, b\} \cup \{a\}$
$= \{0, 1, a, b\}$ and
$A \times B = \{0, 1, b\} \times \{a\} = \{0, a\} \in T_\cup$.

T_s and $T_\cup$ are different as topological spaces.

For A, B $\in T_\cap = \{S' = S \cup \{\phi\}, \cap, +\}$ we see

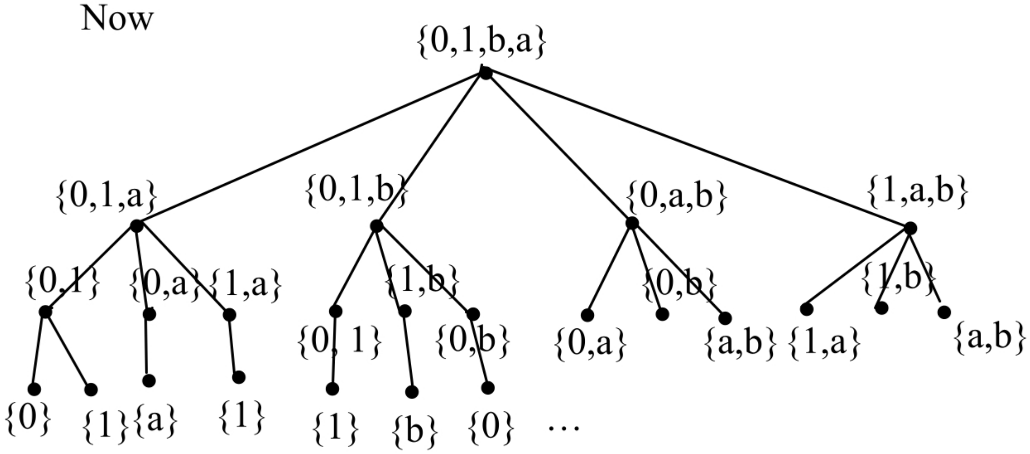
$A + B = \{0, 1, b\} + \{a\} = \{1, a\}$ and
$A \cap B = \{0, 1, b\} + \{a\} = \phi \in T_\cap$.

$T_\cap$ is different from T_s and $T_\cup$ as topological spaces.

Let A, B $\in T_o$
$A \cup B = \{0, 1, b\} \cup \{a\} = \{0, 1, a, b\}$ and
$A \cap B = \{0, 1, b\} \cap \{a\} = \{\phi\} \in T_o$.

T_o is different from $T_\cup$, $T_\cap$ and T_s as topological spaces.

Now



We can draw the trees of $T_\cup$, $T_\cap$, T_o and T_s in more than one way.

Also using the subsemiring P = {a, 1}, find the special types of set ideal semiring topological subset semiring spaces over P. T_o^P, $T_\cap^P$, $T_\cup^P$ and T_S^P are subspaces of T_o, $T_\cap$, $T_\cup$ and T_s.

We suggest the following problems.

Problems

1. Study the four different T_o, $T_\cup$, $T_\cap$ and T_s special subset topological semiring spaces of type I and compare them.

2. Let S = {Collection of all subsets from the ring Z_4} be the subset semiring of type I.

   (i) Find the $o(T_o)$, $o(T_\cup)$, $o(T_\cap)$ and $o(T_s)$.
   (ii) Compare them and show all the four spaces and different.

3. Let S = {Collection of all subsets from the ring $C(Z_4)$} be the subset semiring of type I.

   (i) Study question (i) and (ii) of problem two for this S.
   (ii) Compare S_1 and S.
   (iii) Is $S \subseteq S_1$?
   (iv) Is $T_s \subseteq T_{S_1}$?

4. Let S = {Collection of all subsets from the ring $\langle Z \cup I \rangle$} be the subset semiring of $\langle Z \cup I \rangle$.

   (i) Find T_o, $T_\cup$, $T_\cap$ and T_s and prove all the four subset topological semiring type I spaces are different.
   (ii) Find subset topological semiring type I subspaces of S.

5. Let S_1 = {Collection of all subsets from the ring $\langle R \cup I \rangle$ (g) where $g^2 = 0$} be the subset semiring of type I.

   Study questions (i) to (ii) of problem 4 for this S_1.

   Compare S of problem 4 and S_1 of problem 5.

6. Let S = {Collection of all subsets from the ring $R = C(Z_{12}) \times Z_6 (g_1, g_2, g_3)$ where $g_1^2 = 0$, $g_2^2 = g_2$ and $g_3^2 = -g_3$} be the subset semiring of type I of the ring R.

(i) Find T_o, $T_\cup$, $T_\cap$ and T_{S_2}.

(ii) Find all new subset special topological type I semiring subspaces of T_o, $T_\cup$, $T_\cap$ and T_{S_2}

7. Let S_3 = {Collection of all subsets from the ring $\langle Z_5 \cup I\rangle \times C(Z_6) \times C\langle Z_6 \cup I\rangle)$} be the subset semiring of type I.

   Study questions (i) to (ii) of problem 6 for this S_3.

8. Let S = {Collection of all subsets from the ring QS_3} be the subset semiring.

   (i) Find T_o, $T_\cup$, $T_\cap$ and T_s of S.
   (ii) Find all the subset topological subspaces of T_o, $T_\cup$, $T_\cap$ and T_s of S.

9. Let S = {Collection of all subsets from the ring $Z_3 \times Z_{16}$} be the subset semiring of type I.

   (i) Find T_o, $T_\cup$, $T_\cap$ and T_s.
   (ii) Find $o(T_\cup)$.
   (iii) Find all subrings of $Z_3 \times Z_6$.
   (iv) If P_i are subrings of $Z_3 \times Z_{16}$ (say i=1, 2, …, n) find $T_\cup^{P_i}$, $T_\cap^{P_i}$, $T_o^{P_i}$ and $T_S^{P_i}$ for $1 \leq i \leq n$, the set ideal subset semiring topological spaces of S.
   (v) Find $o(T_\cup^{P_i})$.
   (vi) Will $o(T_\cup) = o(T_\cup^{P_i})$ for some i?
   (vii) Find all subspaces of T_o, $T_\cup$, $T_\cap$ and T_s.
   (viii) Which of the topological spaces T_o (or $T_\cup$ or $T_\cap$ or T_s) has maximum number of subspaces?

10. Let S = {Collection of all subsets from the ring $R = Z_3 \times Z_5 \times Z_7$} be the subset semiring of type I of the ring R.

    Study questions (i) to (viii) of problem 9 for this S.

11. Let S_1 = {Collection of all subsets from the ring $R = \langle Z_{12} \cup I\rangle (g)\ g^2 = 0$} be the subset semiring of type I of the ring R.

    (i) Study questions (i) to (viii) of problem 9 for this S.

    (ii) If in $\langle Z_{12} \cup I\rangle (g)$ is replaced by Z_{12};
    study questions (i) to (viii) of problem 9 for this S.

12. Let S = {Collection of all subsets from the ring Z_3S_3} be the subset semiring of type I of the ring Z_3S_3.

    Study questions (i) to (viii) of problem 9 for this S.

13. Let S = {Collection of all subsets from the ring $C(Z_{11})$} be the subset semiring of type I of the ring $C(Z_{11})$.

    Study questions (i) to (viii) of problem 9 for this S.

14. Let S = {Collection of all subsets from the ring $R = C(Z_{12}) \times \langle Z_7 \cup I\rangle$} be the subset semiring of type I for the ring R.

    Study questions (i) to (viii) of problem 9 for this S.

15. Let S = {Collection of all subsets from the ring $R = C(Z_{15})S_9$} be the subset semiring of type I of the ring R.

    Study questions (i) to (viii) of problem 9 for this S.

16. Let S = {Collection of all subsets from the ring $R = C(\langle Z_{18} \cup I\rangle) (S_3 \times D_{2,9})$} be the subset semiring of type I of R.

    Study questions (i) to (viii) of problem 9 for this S.

17. Let S = {Collection of all subsets of the ring Z_{20}} be the subset semiring of type I of the ring Z_{20}.

(i) Find the subset set ideals of S related to the subring M_1 of $P_1 = \{0, 5, 10, 15\}$

(ii) Find the tree associated with P_1.

(iii) Let $P_2 = \{0, 2, 4, \ldots, 18\} \subseteq Z_{20}$ be the subring of Z_{20}. Let M_2 = {Collection of all subset set ideals of S related to P_2}. Find $T_o^{P_2}, T_\cup^{P_2}$, $T_\cap^{P_2}$ and $T_S^{P_2}$ of M_2.

(iv) Find the tree associated with M_2.

(v) If P_2 is replaced by $P_3 = \{0, 10\} \subseteq Z_{20}$ study $T_o^{P_3}, T_\cup^{P_3}$, $T_S^{P_3}$ and $T_\cap^{P_3}$. Find the tree of $T_S^{P_3}$.

(vi) Compare the trees of each P_i's.

18. Let S = {Collection of all subsets of the ring $Z_{12}S_3$} be the subset semiring of type I.

    Study questions (i) to (vi) of problem 17 for this S.

19. Let S = {Collection of all subsets of the ring $R = (Z_4 \times Z_6)\,(D_{2,9})$} be the subset semiring of type I .

    Study questions (i) to (vi) of problem 17 for this S.

20. Let S = {Collection of all subsets of the ring $R = Z_{12}\,(S_4 \times D_{2,7})$} be the subset semiring of type I.

    Study questions (i) to (vi) of problem 17 for this S.

21. Let S = {Collection of all subsets from the ring $Z(g_1, g_2)$ with $g_1^2 = 0$, $g_2^2 = g_2$, $g_1g_2 = g_2g_1 = 0$} be the subset semiring of type I.

    (i) Show S has infinite collection of subset set semiring ideals and related to each of them; we have four strong subset strong set semiring ideal topological spaces.
    (ii) Find $F_\cup$, $F_\cap$, F_o and F_s for this S.

22. Obtain some special features enjoyed by the four spaces $T_\cup$, $T_\cap$, T_o and T_s of any subset semiring S of type I of a ring.

23. Let S = {Collection of all subsets from the ring Z_{15}} be the subset semiring of type I.

    (i) Find all subrings of Z_{15}.
    (ii) Find all subset subsemirings of S.
    (iii) Find all special new subset set ideals semiring topologies of S related to the subring of Z_{12}.
    (iv) Find all strong special new subset set ideal semiring topologies of the subset semirings of S.
    (v) Find for the subset subsemiring M_1 = {Collection of all subsets of the subring $P_1 = \{0, 4, 8\}$} find $T_o^{M_1}$, $T_\cup^{M_1}$, $T_\cap^{M_1}$ and $T_S^{M_1}$. Find subspaces of these four spaces.

24. Let S = {Collection of all subsets of the ring $C(Z_{18})$} be the subset semiring of $C(Z_{18})$.

    (i) Study problems (i) to (iv) of problem 23 for this S.
    (ii) Let $P_1 = Z_{18}$ be the subring of $C(Z_{18})$. M = {Collection of all subsets of the subring P_1} be the subset subsemiring of S.

    Find $T_o^{M_1}$, $T_\cup^{M_1}$, $T_\cap^{M_1}$ and $T_S^{M_1}$ and compare them.

25. Let S = {Collection of all subsets from the ring of $C\langle Z_6 \cup I\rangle$)} be the subset semiring of type I.

    (i) Study problems (i) to (iv) of problem 23 for this S.
    (ii) Find all subset semirings of S. Using each of these subset subsemirings build the related special strong subset set semiring ideal topological semiring spaces of the subset subsemiring.

26. Let S = {Collection of all subsets from the ring $Z_{12} \times \langle Z_7 \cup I\rangle$} be the subset semiring of type I over $Z_{12} \times \langle Z_7 \cup I\rangle$.

    (i) Study questions (i) to (iv) of problem 24 for this S.
    (ii) Find all subset subsemirings of S.

Relative to each of these subset subsemiring we associate a strong new special subset set semiring ideal topological semiring space relative to each subset subsemiring of S.

27. Let S = {Collection of all subsets of the ring $Z_6 \times \langle C(Z_8) \cup I\rangle S_3$} be the subset semiring of type I.

    (i) Show S is non commutative.
    (ii) Study questions (i) to (ii) of problem 24 for this S.
    (iii) Find all subset subsemirings of S.

    Find all strong special new set semiring ideal subset topological spaces relative to each of the subset subsemirings.

28. Let S = {Collection of all subsets from the ring $C(\langle Z_{15} \cup I\rangle)(S_3 \times D_{29})$} be the subset semiring of type I.

    (i) Study questions (i) to (iv) of problem 23 for this S.
    (ii) Prove S is non commutative.
    (iii) Find all subset subsemiring of S. Relative with each of these subset semirings find the subset special strong set ideal semiring topological spaces over the subset subsemirings.

29. Let S = {Collection of all subsets from the ring $C(\langle Z_{40} \cup I\rangle) A_8$} be the subset semiring of type I.

    (i) Study questions (i) to (iv) of problem 23 for this S.
    (ii) Prove S is non commutative.
    (iii) How many subset subsemiring of S exist?
    (iv) With each of the subset subsemirings find the strong new subset set semiring ideal topological spaces over the subset subsemiring.
    (v) How many subrings of $C(\langle Z_{40} \cup I\rangle)$ exist?
    (vi) Find all the subset special new set semiring ideal topological spaces associated with each of the subrings.

30. Let S = {Collection of all subsets from the ring $Z(g_1, g_2)$; $g_1^2 = 0$, $g_2^2 = g_2$; $g_1g_2 = g_2g_1 = 0$} be the subset semiring.

    Study questions (i) to (vi) of problem 29 for this S.

31. Let S = {Collection of all subsets from the ring $C(Z_{12} \cup I)(g_1,g_2)$ where $g_1^2 = 0$, $g_2^2 = g_2$ and $g_1g_2 = g_2g_1 = 0$} be the subset semiring of type I.

    Study questions (i) to (vi) of problem 29 for this S.

32. Let S = {Collection of all subsets of the ring $C(\langle Z_9 \cup I\rangle)S_4$} be the subset semiring of type I.

    Study questions (i) to (vi) of problem 29 for this S.

33. Let S = {Collection of all subsets of the ring $C(Z_{14})\ A_4 \times D_{2,7}$} be the subset semiring of type I.

    Study questions (i) to (vi) of problem 29 for this S.

34. Let S = {Collection of all subsets from the ring $(Z_4 \times C(Z_{26}))\ (D_{2,6} \times S_4)$} be the subset semiring of type I.

    Study questions (i) to (vi) of problem 29 for this S.

35. Let S = {Collection of all subsets from the ring $C(\langle Z_6 \cup I\rangle)\ S_7$} be the subset semiring of type I.

    Study questions (i) to (vi) of problem 29 for this S.

36. Let S = {Collection of all subsets from the ring $C(Z_9)$} be the subset semiring of type I.

    (i) Find all special new subset semiring topological spaces and the subspaces.

37. Let S = {Collection of all subsets from the ring $C(\langle Z_{11} \cup I\rangle)$} be the subset semiring of type I.

    Study question (i) of problem 36 for this S.

38. Let S = {Collection of all subsets from the semiring $Z^+ \cup \{0\}$} be the subset semiring of type II.

    (i) Find all subset subsemirings of S.
    (ii) Is S a S-subset semiring?
    (iii) Can S contain subset semiring ideals?
    (iv) Can S have subset S-semiring ideals?
    (v) Study the four types of topological spaces associated with S; T_o, $T_\cup$, $T_\cap$ and T_s.

39. Let S = {Collection of all subsets of the semiring $\langle Q^+ \cup I \cup \{0\}\rangle$} be the subset semiring.

    Study questions (i) to (v) of problem 38 for this S.

40. Let S = {Collection of all subsets from the semiring $(Z^+ \cup \{0\})\ (g_1, g_2)$ where $g_1^2 = 0$, $g_2^2 = g_2$} be the subset semiring.

    Study questions (i) to (v) of problem 38 for this S.

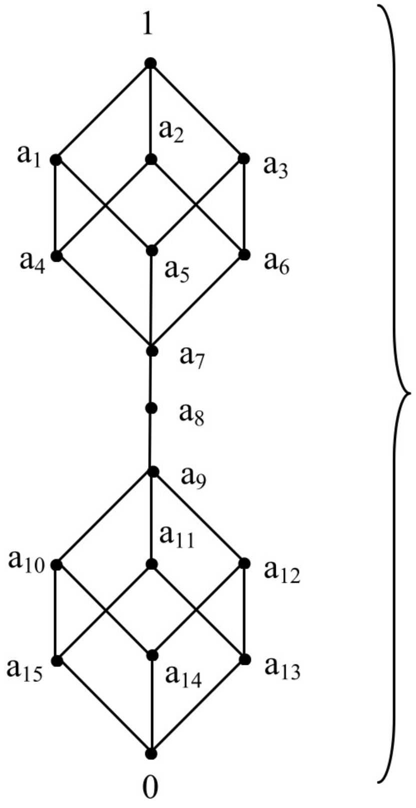
41. Let S = {Collection of all subsets from the semiring $(Z^+ \cup \{0\})\ S_4$} be the subset semiring.

    Study questions (i) to (v) of problem 38 for this S.

42. Collection of all subsets from the semiring $(\langle Q^+ \cup I \cup \{0\})\ S(4)$ be the subset semiring.

    Study questions (i) to (v) of problem 38 for this S.

43. Let S = {Collection of all subsets from the semiring L =



be the subset semiring.

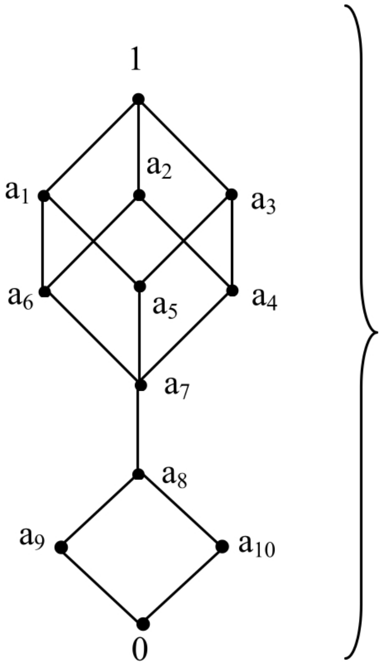
(i) Find o(S).

(ii) Is S a Smarandache subset semiring?

(iii) Find all subset subsemirings of S.

(iv) Can S have special new subset set semiring ideal of finite order?

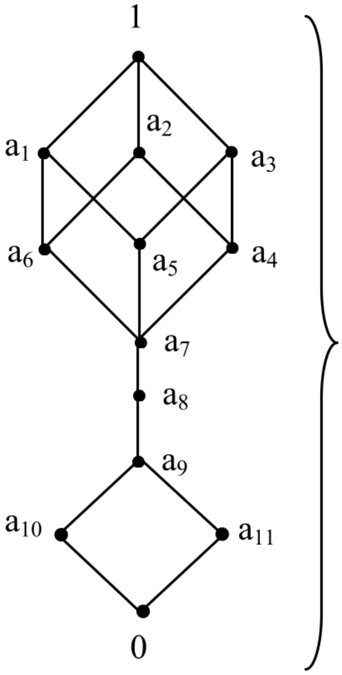
45. Let S = {Collection of all subsets from the semiring LA_3 where L is the lattice



be the subset semiring.

Study questions (i) to (iv) of problem 44 for this S.

46. Let S = {Collection of all subsets from the semiring $L(D_{2,7} \times A_4)$ where L is a lattice given by



be the subset semiring.

Study questions (i) to (iv) of problem 44 for this S.

47. Let S = {Collection of all subsets from the semiring $(L_1 \times L_2)$ $(D_{2,9} \times A_6)$} be the subset semiring of type II.

    Study questions (i) to (iv) of problem 44 for this S.

48. Let S = {Collection of all subsets from the semiring $L(g_1, g_2) = \{a_1 + a_2g_1 + a_3g_2 \mid a_i \in L; 1 \leq i \leq 3;\ g_1^2 = g_1,$ $g_2^2 = 0,\ g_1g_2 = g_2g_1 = 0\}$; where L is a Boolean algebra of order 16} be the subset semiring.

    Study questions (i) to (iv) of problem 44 for this S.

49. Let S = {Collection of all subsets from the semiring $\langle R^+ \cup I\rangle \cup \{0\}$} be the subset semiring.

    (i) Show S has infinite number of subset subsemirings.
    (ii) Show S has infinite number of special new subset topological subspaces of the four types of spaces T_o, $T_\cup$, $T_\cap$ and T_s.
    (iii) Show S has infinite number of special new subset set semiring ideal topological subspaces relative to the subset subsemirings of S.
    (iv) Find subspaces of the above said spaces in (ii)
    (v) Show S has infinite number of strong special new subset set ideal semiring topological spaces over the subset subsemirings.
    (vi) Find strong topological subspaces of the spaces mentioned in (v).

50. Let S = {Collection of all subsets from the semiring $(Z^+ \cup \{0\})S_3$} be the subset semiring of type II.

    (i) Study questions (i) to (vi) of problem 49 for this S.
    (ii) Show S is non commutative.

51. Let S = {Collection of all subsets from the semiring $(\langle R^+ \cup I\rangle \cup \{0\})\,(S_7 \times D_{2,8})$} be the subset semiring of type II.

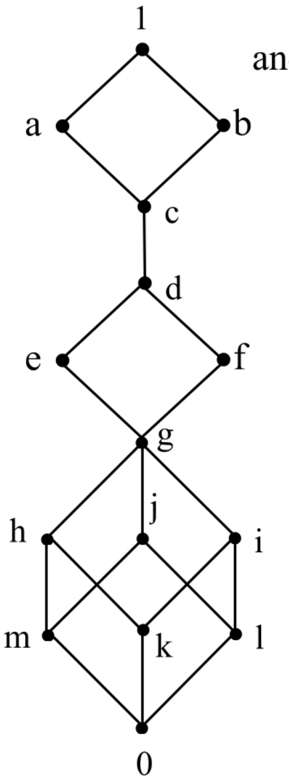
    (i) Study questions (i) to (vi) of problem 49 for this S.
    (ii) Prove S is non commutative.
    (iii) Find all the non commutative subspaces of S.

52. Let S = {Collection of all subsets from the semiring $(Z^+ \cup \{0\})\,(S_7 \times D_{2,4})$} be the subset semiring.

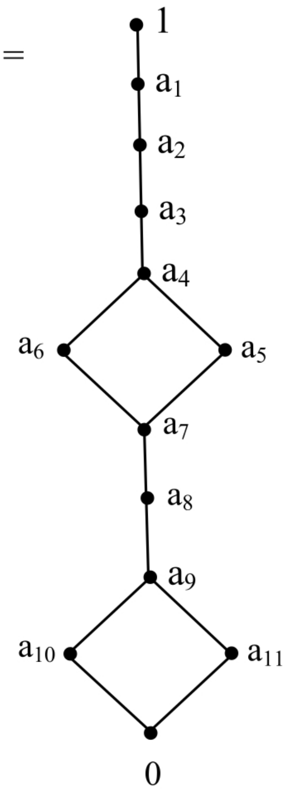
    (i) Prove S is non commutative.
    (ii) Study questions (i) to (vi) of problem 49 for this S.

53. Let S = {Collection of all subsets from the semiring $(L_1 \times L_2)\,(S_3 \times S(4))$} where

$L_1 =$



and $L_2 =$



be the subset semiring.

(i) Prove S is a non commutative subset semiring.

(ii) Find all subset subsemirings of S.

(iii) Find all types of subset special new topological semiring spaces of S.

(iv) Find all four types of special strong new topological set semiring ideal spaces of S related to every subset subsemirings.

(v) Find all special strong new topological set semiring ideal subspaces of S related to every subset subsemiring of S.

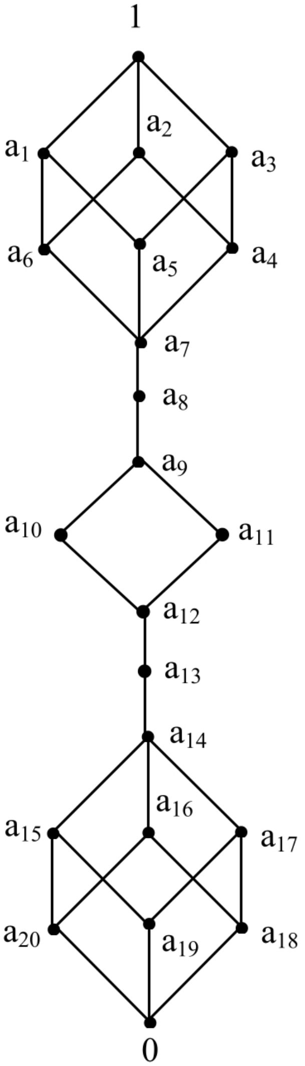
(vi) Find the total number of subset topological subspaces of T_o, $T_\cup$, $T_\cap$ and T_s.

54. Find some interesting and distinct features associated with the four types of topological spaces T_o, $T_\cup$, $T_\cap$ and T_s.

55. What can one say about the non commutative new special set ideal semiring topological subset semiring spaces?

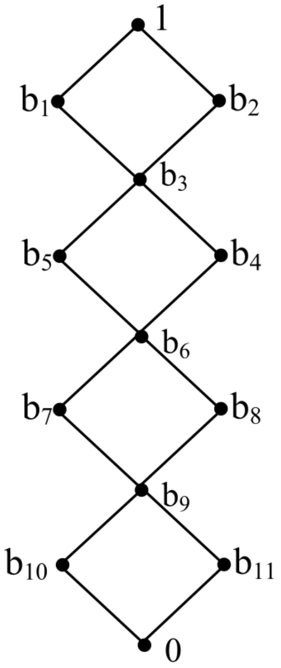
56. Compare the topological spaces associated with type I subset semirings with the type II subset semirings.

57. Let S = {Collection of all subsets from the semiring $L(S(3) \times A_4 \times D_{2,7})$} be the subset semiring of type II where L is a lattice given in the following:



Study questions (i) to (vi) of problem 53 for this S.

58. Let S = {Collection of all subsets from the semiring $(L_1 \times L_2 \times L_3)$ $(A_5 \times D_{2,7})$} be the subset semiring of type I where L_1 is a chain lattice C_{20}, L_2 is a Boolean algebra of order 32 and L_3 =



Study questions (i) to (vi) of problem 53 for this S.

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